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Influence of construction method on the load bearing capacity of skew masonry arches

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ABSTRACT

This paper investigates the influence of construction method (e.g. false skew, helicoidal and logarithmic method) on the mechanical behaviour and load carrying capacity of single span skew masonry arches. Simulations were performed with the three dimensional computational software, based on the Discrete Element Method of analysis. Each stone/brick of the masonry skew arch was represented by a distinct block. Mortar joints were modelled as zero thickness interfaces which can open and close depending on the magnitude and direction of the stresses applied to them. The variables investigated were the construction method, the angle of skew, the size of masonry blocks and the critical location of the live load along the span of the arch. At each skew arch, a full width vertical line load was applied incrementally until collapse. From the results analysis, it was found that for a skew masonry arch constructed using the false skew method, as the angle of skew increase, sliding between voussoirs in the arch increases and failure load decreases. However, for skew masonry arches constructed using the helicoidal and logarithmic method, as the angle of skew increases, the failure load increases. Three different characteristic failure modes were identified depending on the contact friction angle and the method of construction. These observations provide new insight into the behaviour and lead to suggestions for understanding the load carrying capacity and failure mode of skew arches.

1. Introduction

Masonry arch bridges have been used for at least four millennia. In Europe alone, there are thousands of masonry arch bridges which still form part of the highway and railway networks. As a result of the over-conservative design methods used for their construction in the past, these bridges are usually able to carry the even-increasing live loads from modern day traffic. Also, masonry arch bridges have been proved to be an extremely durable structural form and considered to be aesthetically pleasing. However, most of these bridges are now old and are deteriorating over time [1]. Moreover, the different materials and methods of construction (e.g. masonry skew arches which enable to span obstacles at an angle) used in these bridges will influence their strength and stiffness [2,3].

Over the last two decades, considerable effort has been put into gaining a greater understanding of the behaviour of masonry arch bridges with a view to improve resilience of transport corridors and efficiency when assessing the serviceability and ultimate limit state behaviour of such bridges [2]. However, many approaches (e.g. analytical methods and numerical techniques) used for the assessment of masonry arches has been recognised as being highly conservative and predict collapse loads far lower than predicted by experience [2]. Furthermore, although it is well understood that masonry arch bridges behave in a three dimensional manner [3–5] a great deal of work has been carried out to assess the strength of square span masonry arch bridges using mainly two dimensional methods of analysis [6–9]. For example, in UK as well as other parts of Europe, the most commonly used method for the assessment of masonry arch bridges in the industry is the “MEXE”. This is a semi-empirical approach based on an elastic analysis by Pippard et al. [10] who modelled the arch barrel as linear elastic, segmental in shape, pinned at its support and carrying a central point load. Although the approach is quick and easy to use, it has been found to be over-conservative and in some cases highly subjective to parameters used for the estimation of the maximum load. Other approaches used by the industry are mostly based on 2D limit analysis. The static theorem of plastic limit analysis uses simple equilibrium calculations: the self-weigh of the arch barrel and live loads are tried to be balanced by forces between the blocks. If any equilibrated force system can be found, then the structure is safe. The kinematic theorem of limit analysis identifies a collapse state with possibly smallest

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Nomenclature

Ω	angle of skew
R	mid-radius of the arch barrel
s	span of the arch
b	breadth of the arch (parallel to the abutments)
t	barrel thickness
L	length of the voussoir
W	width of the voussoir
x	loading position (measured from the abutments)
K_n	contact elastic normal stiffness
K_s	contact elastic shear stiffness
φ	internal friction of the contact
φ_{res}	residual internal friction of contact
c	cohesion of the contact
c_{res}	residual cohesion of the contact
f_t	tensile strength of the contact
ψ	dilatation angle of contact
ρ	density of the blocks
σ	normal stress
τ	shear stress

external loading and hence predict the ultimate load, in such a way that if any mechanism can be found on which the loads make positive work, then the loads will cause collapse. Using the robustness of linear programming ultimate load bearing capacity and the failure mode can be estimated. However, the two types of solutions for the failure load are not necessarily the same. In linear programming the primal and the dual problem lead to coinciding solution only if the optimization problem is convex; in mechanical sense it means that the flow rule in the system must be associated. For structures consisting of rigid masonry blocks the associativity means that the contact friction angle should to be equal to the contact dilation angle, which would be a rather unrealistic model of reality. In case of non-associated flow rule, on the other hand, the behaviour becomes history-dependent, and the two theorems do not give the same result. (The issue has already been mentioned in [31], then appeared in several limit analysis masonry papers.) This duality gap justifies those efforts that attempt to find history-simulating methods for masonry analysis, particularly for problems where frictional sliding may become crucial. The discrete element method (DEM) that was used in the present paper is an advantageous possibility for this.

With the use of sophisticated methods of analysis like Finite Element Method (FEM), efforts have been made to understand the three dimensional behaviour of arches (e.g. [11–14]). The disadvantages of the finite element method are mainly associated with: (a) relatively high computational cost in comparison to limit analysis; (b) crack development cannot be obtained without a priori knowledge of where to expect cracks; and (c) convergence difficulties if blocks fall or slide excessively. An alternative and appealing approach is represented by the Distinct Element Method (DEM), where the discrete nature of the masonry arch is truly incorporated [15]. The advantage of the DEM is that it considers the arch as a collection of separate voussoirs able to move and rotate relative to each other; the main disadvantage is the often huge computational cost, even if compared to FEM [16]. The DEM was initially developed by Cundall ([17]) to model blocky-rock systems and sliding of individual pieces of stones along joints. The approach was later used to model masonry structures including arches ([3,18,19,25,29,30]), where failure occurs along mortar joints. These studies demonstrated that DEM is a suitable method to perform analysis of masonry arches and to describe realistically the ultimate load and failure mechanism.

Skew arches, i.e. arches that span obstacles not perpendicularly but at an angle, are most common in areas rich in rivulets or valleys of varying directions, e.g. around delta firths. During the industrial revolution the number of skew arches started to increase rapidly, since the straightness of the railway track was one of the most important aspect in the course of the design. Three main techniques were suggested for their construction in the 19th Century: (1) the false skew arch; (2) the helicoidal method; and (3) the logarithmic method (see their description below in Section 3, or in more detail in Forgács et al. [3]). Fig. 1 shows stone masonry skew arch bridges constructed using different construction methods.

Over the last 30 years, although significant research work has been carried out to understand the mechanical behaviour of arches, limited research been carried out to understand the mechanical behaviour of skew arches. Abdunur [20] investigated a shallow, 45° helicoidal brickwork skew arch consisting of two rings. The arch was loaded to collapse with a line load parallel to the abutments. Later, Wang [5] tested a similar, but a more narrow skew arch subjected to a patch load at quarter span. The arch failed due to the formation of hinged mechanism. The hinges were parallel to the abutments, and this was mainly due to the stiffening effect of internal spandrel walls and masonry backing. In addition, Hendry et al. [21] and Melbourne [9] investigated a 16° skewed semi-circular arch masonry bridge with backfill and spandrel walls. Failure was due to a compressive failure of the arch beneath the loading position. Moreover, Sarhosis et al. [19] investigated numerically the influence of skew angle on segmental and circular skew arches constructed with joint parallel to springing. The behaviour of skew arches compared against those of "square" or regular arches. It was found that an increase in the angle of skew will increase the twisting behaviour of the arch and will cause failure to occur at a lower load. Also, the effect of angle of skew on the ultimate load is more significant for segmental rather than circular arches. In previous work, the present authors investigated the minimum necessary thickness of single-span skew masonry arches carrying their self-weight only [3]. It was found that the construction method and the angle of skew significantly influences the minimum barrel thickness of the arch. For skew arches constructed using the false method, as the angle of skew increases, the minimum barrel thickness increases. However, for skew arches constructed using the helicoidal and logarithmic method, as the angle of skew increases, the minimum barrel thickness decreases. In contrast to rectangular arches, the size of the masonry units and the joint friction angle significantly influences the mechanical behaviour of skewed masonry arches.

The aim of this paper is to extend the previous study carried out by the authors [3] and investigate the influence of construction method on the load carrying capacity of skew masonry arches. It is anticipated that such results will provide useful information and guidance to design engineers and practitioners. Using the three-dimensional DEM software [22], computational models were developed to predict the ultimate behaviour of different in construction masonry skew arches with different skew angles. DEM is well suited for collapse analysis of stone masonry structures since: (a) large displacements and rotations between blocks, including their partial or complete detachment, can be simulated; (b) contacts between blocks are automatically detected and updated as block motion occurs; and (c) progressive failure associated with crack propagation can be simulated.

At the present study, the false skew, the logarithmic, and the helicoidal method of construction of skew arches have been studied. According to [1,23] the rise-to-span ratio of masonry bridge stock varies significantly from country to country. However, from [1,23] it was found that the most common shape for skew masonry arches is that of a semi-circular (e.g. 50% of masonry bridges on Italian railways, 92% of Hungarian masonry bridges). Therefore, as part of this study, semi-circular arches (with rise-to-span ratio: 1:2) were investigated. In addition, since the intention of the authors was to investigate the effect of