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Nonlinear modeling of MDOF structures equipped with viscoelastic dampers with strain, temperature and frequency-dependent properties

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ABSTRACT

The main objective of this paper is to propose a numerical algorithm for modeling the nonlinear time-history seismic response of MDOF structures with frequency-, strain- and temperature-dependent viscoelastic (VE) dampers. An extended recursive parameter model based on Nakamura's method is established in order to simulate the frequency-, strain- and temperature-dependent properties of VE dampers in the time domain. The numerical stability of the extended model combined with a time-stepping method is studied and the model is verified against the exact solution for linear response as well as the experimental measurements for nonlinear response. A full-fledged nonlinear time-history analysis of a practical engineering problem is carried out and the results are compared with those calculated by traditional Kelvin-Voigt model which is commonly used in the practice as well as fractional derivative (FD) method. The advantages and robustness of the proposed method is then fully discussed.

1. Introduction

Viscoelastic (VE) dampers have been widely used in Civil Engineering structures to reduce excessive wind- or seismic-induced vibrations, some of which are reported by Mahmoodi et al. [1], Soong and Dargush [2], Symans and Constantinou [3] and Christopoulos and Montgomery [4], Hejazi et al. ([5] & [6]) and Kang and Tagawa [7].

VE dampers are known to have the frequency-, strain- and temperature-dependent properties which were measured through experimental investigations as presented by Makris and Constantinou [8], Kasai et al. [9] and [10], Chang et al. [11] and Xue et al. [12]. On this basis, the constitutive model of a VE damper, which relates the strain (or displacement) to the stress (or force), is complex and formulated in frequency domain for specified strain and temperature levels.

Numerical modeling of structural systems coupled with VE dampers subjected to dynamic loading requires the incorporation of the strain-, frequency- and temperature-dependent impedance functions into the mathematical model. The most straightforward method is to formulate and solve the governing equations in the frequency domain. This can be justified when the structure behaves in linear elastic range, low strain amplitude is expected for the VE damper and the effect of temperature-rise inside VE damper, which is dependent on strain amplitude (Kasai et al. [10]), is insignificant. However, when either an inelastic hysteretic behavior of the structure is expected or the strain- and temperature-dependent behavior of VE material is of concern, this method lacks accuracy because the principal of superposition, on which the frequency domain analysis is established, is not valid anymore.

In order to numerically simulate the coupled system of VE dampers and the structures attributed with nonlinear material behavior under strong ground motions, performing a full-fledged nonlinear dynamic time-history analysis is necessary and the constitutive model of the damper in the frequency domain should be transferred into the time domain, since there is still no comprehensive nonlinear frequency domain methodology available to tackle this problem, especially for the nonlinear structural systems such as tall buildings designed for highly seismic region [4].

An effective approach to this type of modeling is the recursive-parameter model. This method requires dynamic impedance functions *a priori*, which can be obtained from experiments. The method is applicable to an MDOF system coupled with VE dampers at different locations. However, special numerical treatment is required if the impedance function is also amplitude and temperature dependent. The main objective of this study is to propose a method by which the nonlinear time-history response of an inelastic hysteretic MDOF system coupled with a frequency, strain and temperature dependent VE damper is assessed through a combination of recursive-parameter model and time-integration method.

The novelty in the current study stems from the fact that the

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recursive parameter modeling technique is extended and applied to the abovementioned case study. A methodology for incorporating the strain- and temperature-dependent properties into the recursive parameter model is suggested and verified which is able to update the recursive-parameter model coefficients at each time-step of the analysis based on the state of the VE material.

In addition, a few other approaches including the model based on temporal discretization of fractional derivative (FD) formulation using weight functions and the classical Kelvin-Voigt model, widely used in the engineering practice, are employed in this study. A detailed comparison among the introduced methods is made in order to highlight the efficacy and robustness of the proposed model for the application in different case studies.

In Section 2 of this paper, the mathematical constitutive models for VE dampers are briefly discussed and several recursive-parameter models are reviewed to provide a general background on time-domain presentation of frequency dependent impedance functions. Based on the mathematical principals for the modeling of VE material, an extended recursive-parameter model is suggested which is able to account for temperature- and strain-dependent properties. Section 3 presents the mathematical formulation of the governing equations and the solution strategy for presented nonlinear equations in the time-domain. In Section 4, the stability analysis results are illustrated and the proposed methodology is verified against different methods. Section 5 summarizes the main findings and results on the applications of the proposed method to some practical engineering cases. Finally, the main conclusions of the research are presented in Section 6.

2. Mathematical models for simulating ve damper behavior

Different approaches for the mathematical modeling of the dynamic behavior of VE dampers are available in the literature ([13] and [14]). In the classical linear mechanical model, the VE damper is simulated using a combination of linear springs and dashpots. Standard Kelvin-Voigt and Maxwell models are derived from the linear model where the former represents the VE damper by spring and dashpot elements in parallel and the latter comprises of a springs and dashpots connected in series. The constitutive law, which represents the stress-strain (or force-displacement) relation, is expressed in a differential operator form.

By replacing the regular differential operators with fractional-order differential operators, a more robust approach is derived which has been used by many researches and is referred to as the FD method. The main advantage of this method originates from its ability to cover the broad-band behavior of viscoelastic materials with a small number of parameters both in the time domain and the frequency domain. The generic form of the VE damper constitutive equation can be expressed as:

$$R(t) = \frac{GA_s}{d}[\{u(t)\} + bD^{\alpha}\{u(t)\}] \quad (1)$$

where $D^{\alpha}f(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha}}d\tau$ is the α-order fractional derivative of Riemann-Liouville integral, $\Gamma()$ is the gamma function [15], R is the resisting force of VE damper and u is the VE damper deformation. The other parameters in the above equations are defined as follows:

G: shear modulus of the VE material
b and α: material constants
A_s: shear area of the VE material
D: thickness of the VE material

Frequency domain representation of Eq. (1) results in the following expressions for amplitude- and temperature-independent VE materials:

$$\frac{R(iw)}{u(iw)} = K_d(\omega) + i\omega C_d(\omega)$$

$$K_d(\omega) = G\frac{1 + b\omega^{\alpha}\cos(\frac{\alpha\pi}{2})}{1 + 2\alpha\omega^{\alpha}\cos(\frac{\alpha\pi}{2})}\frac{A_s}{d}$$

$$C_d(\omega) = \frac{K_d(\omega)\eta_d(\omega)}{\omega} = \frac{K_d(\omega)}{\omega}\frac{b\omega^{\alpha}\sin(\frac{\alpha\pi}{2})}{1 + b\omega^{\alpha}\cos(\frac{\alpha\pi}{2})} \quad (2)$$

where $K_d(\omega)$ and $C_d(\omega)$ are the frequency-dependent stiffness and damping of VE damper. η_d is defined as the loss factor of VE material. The constants in Eq. (2) can be experimentally calibrated from the tests in the frequency domain such as sinusoidal tests for a specific strain amplitude and temperature level.

In the time domain analysis, the behavior of VE damper can be simulated by either discrete-time presentation of Eq. (1) using weight functions or time representation of frequency-dependent stiffness and damping using recursive parameter models. The latter method can be applied to a wide range of frequency-dependent physical system with arbitrary stiffness and damping functions. The reaction force of VE damper in both cases is expressed by a recursive equation formulated in terms of current and previous response variables.Assuming linear weight functions [16] or quadratic ones [17] the generic time-discrete form of the Eq. (1) can be derived as:

$$R(t) = \frac{GA_s}{d}\sum_{j=0}^{N-1} k_j u(t-j\Delta t) + \sum_{j=1}^{N-1} r_j R(t-j\Delta t) \quad (3)$$

The methodology for calculating coefficients k_j and r_j in the above equation, based on linear or quadratic distribution of shear strain between two time steps, are given in Refs. [16] and [17] and are not reproduced here.

Different recursive parameter models are available in the literature to replicate the frequency dependency of an impedance function in time domain analysis.

In the classic approach, the reaction force is only a function of previous step displacement. This methodology has been followed by several researchers in the past, such as the ones proposed by Wolf and Obernhuber [18] and Zhang et al. [19].

As another approach, the interaction forces at a specified time can be expressed in terms of the displacements at the same time and the most recent forces and the most recent past displacements similar to the one shown by Eq. (3). This method has been employed in a few research studies, some of which are reported by Wolf and Motosaka [20], Paronesso and Wolf [21], Safak [22] and Okada et al. [23].

Special numerical treatments might be required for the above methods to guarantee the numerical stability, which makes them computationally demanding in some cases.

Recently, a numerically robust and stable approach has been proposed by Nakamura [24] in which the past displacement, velocity and acceleration are included in the recursive formulation. The recursive coefficients are computed by solving a linear system of equations resulted from equating the analytical impedance function with the approximate one estimated by the recursive equation for a number of frequency data points.

The application of these methods can be extended to the MDOF systems equipped with VE dampers with known impedance functions. In general, the interface force from a frequency-dependent system can be presented as a function of previous time steps' displacement, velocity, acceleration or even reaction force histories, depending on the selected method.

In reality, the behavior of VE material is not linear and several sources of nonlinear behavior can arise. Kasai et al. [10] detected four shapes of nonlinearity through dynamic experimental investigations: (a) softening by temperature increase (b) softening by large strain (c) hardening by high strain-rate (d) hardening by large strain.

Here, we propose an extended recursive parameter model, based on Nakamura's model [24], in order to account for both strain- and temperature-dependent properties as follows: