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Design sensitivity analysis for optimal design of geometrically nonlinear lattice structures

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ABSTRACT

Lattice structures are quite sensitive to the lateral wind loading and prone to the nonlinear behavior and lateral buckling. However, research on the design sensitivity analysis for lattice structures accounting for the geometric nonlinearity is rarely reported in the literature. An approach is proposed in this paper for the design sensitivity analysis for the nodal displacements, element stresses and critical load factor of linear and nonlinear lattice structures. For this, a design sensitivity formula of the global internal force vector with respect to design variables is first derived. A three-bar planar truss is then used to verify the proposed sensitivity analysis formula for the global internal force and the corresponding displacements and stresses. The ability of the proposed approach in estimating the critical load factor and computing its design sensibility is also proposed and compared with those results reported in the literature for a 3D roof structure. Good agreements between the results of the proposed approach with theoretical counterparts and those reported in the literature demonstrate that the proposed approach is effective and accurate. Finally, a real 3D lattice structure is adopted to investigate the application and effectiveness of the proposed approach for the design sensitivity analysis on the structure under wind actions. Results of design sensitivity for the lattice structure under wind actions show that the proposed approach could be used to estimate the design sensitivities effectively for both linear and nonlinear structural systems.

1. Introduction

Lattice structures have inherent characteristics in high flexibility, lightweight and small damping, thus they are sensitive to the lateral wind loading. It is desirable that the serviceability and strength in lattice structures should be satisfactory and at the same time, their weight and cost could be minimized. Therefore it is necessary to conduct the wind-induced response analysis correctly and to achieve the wind-resistant optimization design for the lattice structures. The elements of lattice structures have small cross-sections and are quite slender. Hence, this type of structures is usually very flexible and prone to the geometric nonlinear behaviour and instability.

In the meantime, the optimization design process for such structures is time-consuming. Although great advances have been made in automating the design process [1], an effective structural optimization method is not available for such nonlinear structural systems under dynamic wind actions [2]. There are two major categories of approaches for structural optimization analyses. The first category mainly includes discrete structural optimization method. At the beginning of the 1980s, several stochastic approaches to discrete structural optimization were introduced. The best known of these are genetic algorithm; simulated annealing; evolutionary optimization. A very simple and robust algorithm for finding the (near) minimum weight of a structure composed of elements assigned from a finite list of available parameters had been presented using discrete structural optimization by removing redundant material [3]. A design procedure employing a Teaching–Learning Based Optimization (TLBO) technique for discrete optimization of planar steel frames was proposed for the problem of engineering design applications [4]. Other meta heuristics optimization methods, such as random search method [5], harmony search method [6] and Artificial Bee Colony (ABC) optimization method [7] have been suggested in the literature and new algorithms are published on a regular basis. A critical review was conducted for truss optimization with discrete design variables [8].

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Nomenclature

$\{f\}$	element internal force vector in local coordinate system
$\{f\}_i$	internal force vector in local coordinate system for the *i*-th element
$\{f\}_k^i$	internal node vector in local coordinate system at left node *i* of the *k*-th element
$\{f\}_k^j$	internal node vector in local coordinate system at right node *j* of the *k*-th element
$\{\sigma\}$	element stress vector
$\{\sigma\}_k^i$	stress vector in local coordinate system at the location of node *i* for the *k*-th element
$\{\sigma\}_k^j$	stress vector in local coordinate system at the location of node *j* for the *k*-th element
$\{F\}$	element internal force vector in global coordinate system
$\{F\}_i$	internal force vector in global coordinate system for the *i*-th element
$\{d\}$	element displacement vector in the global coordinate system
$\{d\}_i$	displacement vector in the global coordinate system for the *i*-th element
$[T]$	transformation matrix between local and global coordinate systems
$[T]_i$	transformation matrix between local and global coordinate systems for the *i*-th element
$[\mathrm{k_T}]$	element tangent stiffness matrix
${}^t[\mathrm{k_T}]_i$	element tangent stiffness matrix for the *i*-th element at time step *t*
$[\mathrm{k_L}]$	element linear elastic stiffness matrix
$[\mathrm{k_L}]_i$	element linear elastic stiffness matrix for the *i*-th element at time step *t*
$[\mathrm{k_d}]$	element large displacement matrix
${}^t[\mathrm{k_d}]_i$	element large displacement matrix for the *i*-th element at time step *t*
$[\mathrm{k}_\sigma]$	element initial stress matrix
${}^t[\mathrm{k}_\sigma]_i$	element initial stress matrix for the *i*-th element at time step *t*
$\{P\}_{n\times 1}$	global internal force vector for all structural nodes
$\{U\}$	global displacement for all structural nodes
$\{b\}$	vector of design variables
$[\mathrm{K}]$	global tangent stiffness matrix for the structure
${}^t[K]$, ${}^{t\text{-}\Delta t}[K]$	global tangent stiffness for two states at time steps t ant t-Δt
${}^c[K]$	tangent stiffness at the critical buckling point
$\{R\}$	external load vector
${}^0\{R\}$	reference load vector
${}^c\{R\}$	estimated critical load
${}^t\lambda$	loading coefficient
${}^c\lambda$	estimated critical load factor
${}^t\mu$	loading coefficient
${}^t\gamma_1$	solution of the minimum eigenvalue for Eq. (21)

The other category mainly include continuous structural optimization method which including Mathematical Programming (MP) techniques and Optimality Criteria (OC) [9]. The OC approach has gained tremendous popularity since it is particularly suitable for large-scale structures with weak dependence on the size of the structure or the number of design variables [10].

The accurate derivatives (gradient) of constraints and optimization functions to design variables are essential for the OC method; Chan [11] developed a virtual work formulation approach for the lateral stiffness and wind resistant design of tall buildings. The design sensitivities of lateral displacements, inter-story drifts and the natural frequency were derived with the help of the principle of virtual work and Rayleigh method. The design sensitivity of lateral displacements in tall building under lateral wind loading were obtained by the following formula:

$$\frac{\partial\Delta}{\partial b_k} \approx \sum_{ne}\left(\int_0^l\left(\frac{\overline{\overline{N}}N_P}{E}+\frac{\eta\overline{\overline{Q}}Q_P}{G}\right)\frac{\partial(A^{-1})}{\partial b_k}+\frac{\overline{\overline{M}}M_P}{E}\frac{\partial(I^{-1})}{\partial b_k}dx\right) \tag{1}$$

where Δ,b_k,A,I,E,G,η,ne are lateral displacement, the *k*-th design variable, cross section, inertia moment, elastic modulus, shear modulus, non-uniform coefficient for shear force and total number of elements respectively. $\overline{N}$,$\overline{Q}$,$\overline{M}$ are calculated axial force, shear force and bending moment for each element when a unit load is applied at the optimized structure along the direction of lateral displacement constrained. N_P,Q_P,M_P are calculated axial force, shear force and bending moment for each element when the current wind loadings are applied at the optimized structure. The above approach assumes that the internal forces of the structural elements are independent of deformations of a structure and so remain unchanged within the optimization process. The strain energy method was ever applied for the nonlinear structural systems by Pezeshk [12] and by Saka and Kameshki [13]. However this method need to calculate the internal forces and deformations in two load cases: one under current loads and the other under unit loads, they are time-consuming.

Meanwhile different approaches to obtain the explicit formulation of design sensitivity for the displacement, stress and natural frequency for linear structural systems were proposed by Cheng and Truman [14] using information of element and/or global stiffness matrices, which are not normally provided by many finite element programs. There are a number of investigations on the accuracy and effectiveness of this approach in estimating the design sensitivities of the displacement, stress and natural frequency for linear structural systems. However, research focused on the design sensitivity analysis on the nonlinear structural system using this method is rarely reported in the literature.

As the high-rise lattice structures demonstrate the nonlinear structural behaviour under wind loading, geometrical nonlinear analysis is normally needed for the structural analysis for this type of structures. In addition to the normal strength and drift constraint, the lattice structure should be designed to be capable of resisting the nonlinear behaviour and buckling.

The nonlinear analysis and design optimization of guyed masts were reported by Heydari et al. [15]. The sensitivities of the objective function, displacement and strength constraints were carried out in a semi-analytical form in their work. A procedure for the sensitivity analysis by a nonlinear incremental-iterative structural analysis of frames was also proposed by Wang and Chan [16], Wu and Arora [17].

Compared with the design sensitivities of displacements and stresses, the sensitivity analysis of the nonlinear critical load factor is more complicated [18–20]. A technique for approximately estimating the design sensitivity of the critical load was presented by Kwon et al. [21]. They proposed a technique for estimating design sensitivities at pre-buckling points by applying “one- or two-point approximation” method for the critical load estimation. Design optimization of geometric and material nonlinear problems was studied by Wu [22], and Wu and Arora [23]. It was shown that the explicit formula for the design sensitivity analysis of nonlinear structural systems with 3D beam elements is much more complicated [22].

The design sensitivity analysis of a nonlinear structural system normally requires information of the element and/or global stiffness matrices, particularly the tangent stiffness matrix. For some special structure such as space frame system, the explicit formula of the geometrical nonlinear tangent stiffness matrix for truss elements and 3D beam elements were derived [24–26].

In this paper, analytical formulas for the sensitivity of the global internal force vector and for computation of sensitivities of the