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Dynamic eccentricities and the “capable near collapse centre of stiffness” of reinforced concrete single-storey buildings in pushover analysis

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ABSTRACT

In order to seismically assess an asymmetric single-storey reinforced concrete (r/c) building, the basic method of analysis proposed by all contemporary seismic codes is static nonlinear analysis (pushover). During the application of pushover analysis, two questions arise: (1) which is the point in the plan where the lateral static floor force must be applied? (2) which is the horizontal orientation of this lateral force? According to Eurocode EN1998-1, the point (in the plan) of application of the lateral static floor force is the position of the concentrated translational mass *m* of the diaphragm-floor, which is the mass that has been moved by the accidental floor eccentricity. However, in this way, the diaphragm seismic inertial moment (around Z-axis) is neglected, both in the linear and in the nonlinear area of analysis. Furthermore, Eurocode EN1998-1 defines that the lateral static floor force must be oriented along the appropriate horizontal directions, which means along the building’s horizontal principal axes, according to the international literature. Consequently, during the pushover analysis, the two key issues and main objective of this work are to identify: firstly, the suitable location (in the plan) of the point where the lateral static floor force must be applied and, secondly, the orientation of this force. In order to apply the pushover analysis on asymmetric single-storey r/c buildings aiming at the Near Collapse limit state in a documented manner, an extended parametric analysis is performed from which the following results are obtained: (a) the origin point for the measurement of the relevant dynamic eccentricities, (b) the correct orientation of the lateral static floor force and (c) the magnitude of the relevant dynamic eccentricities. The abovementioned dynamic eccentricities are presented here and related graphs and relationships are provided. In the present paper, the origin point and the appropriate orientations are referred to using the new terms “Capable Near Collapse Centre of Stiffness” CR_{sec} and horizontal “Capable Near Collapse Principal Axes” I_{sec} or II_{sec}.

1. Introduction

From aftershock observations, it has been shown that the corner or perimeter columns of asymmetric buildings present significantly greater damage rates than the internal columns. This is mainly attributed to the existence of the torsional-translational vibrations of the floor diaphragms. Indeed, due to the diaphragm rotation around a vertical axis, additional horizontal displacements occur at the external building outline, especially at the corners. The limitation of the abovementioned additional displacements is achieved by increasing the “total torsional stiffness” of the building, so that the translational vibrations dominate over the torsional ones (about the vertical axis) under pure translational seismic excitation at the building base, and this happens when the fundamental uncoupled torsional frequency is greater than the fundamental translational frequencies along the principal building directions. In the opposite case, when the torsional vibrations dominate over the translational ones, the building is characterized as “torsionally sensitive” and therefore, according to seismic codes, as “irregular in plan” (Eq. (4.1b) & § 4.3.3.1 (8d) & (9)/ EN 1998-1, [1]).

In order to assess a building’s seismic capacity, the basic analysis method that is proposed by all contemporary seismic codes is nonlinear static analysis (pushover). However, when applying pushover analysis some questions must be clarified, two of which are the following:

(a) Which is the suitable point in the plan where the lateral static force of each floor must be applied?
(b) Which is the horizontal orientation of this lateral static floor force?

As regards the first question, Eurocode EN 1998-1 in Section 4.3.3.4.2.2(2)P states that the point of application in the plan of the lateral monotonic increasing static floor force is the position of the concentrated translational mass *m* of the floor diaphragm, which has been moved by the accidental floor eccentricity $e_{ai} = \pm(0.05 \text{ or } 0.10) \cdot L_i$,

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where L_i is the normal floor dimension in relation to the direction of the seismic component.

Concerning the second question, Eurocode EN 1998-1 in Section 4.3.3.4.2.1(3) states that, in "regular in plan" buildings, two planar models can be used, one for every principal direction. In other words, the lateral static floor force must be oriented along one horizontal principal direction of the building. Indeed, the EN 1998-1 in Section 4.3.1(5) states that when the building is "regular in plan", the two planar models have the orientation of the two horizontal principal axes of the building; the same is supported in Section 4.2.1.3(2). Similarly, EN 1998-1 in Section 4.3.3.2.3(2)P states that, in the elevation of the building, the distribution of lateral static floor forces is oriented along the two planar models of the building, and in Section 4.3.3.4.2.2(1) it is added that the above in elevation distribution must be applied when the modal-pattern's vertical distribution of the lateral static floor loads is used in pushover analysis. Also, EN 1998-1 in Sections 4.3.3.1(7), (8) & (9) on linear analysis, once again refers to the horizontal principal directions and in Section 4.3.3.1(11)P states that if all vertical elements of the building are oriented along two orthogonal directions, then these directions are the appropriate (principal) horizontal directions along which the lateral static floor force must be oriented. The problem with Eurocode EN 1998-1 is firstly that information is given only for the determination of the Centre of Stiffness and the torsional radius in single-storey buildings and, in certain cases only, in multi-storey buildings (EN 1998-1 Section 4.2.3.2(7) & (8) &(9)); secondly, that in the case of multi-storey buildings, EN 1998-1 refers us to the National Annexes for special guidelines. The Hellenic National Annex of EN 1998-1 refers to the international literature as regards multi-storey buildings and, in the linear area of analysis, imposes the use of a fictitious vertical elastic axis (an axis that intersects the plan of the floors at the building's fictitious Centre of Stiffness) and fictitious horizontal principal axes [2–4]. The abovementioned fictitious elastic axes are proposed because there is a great deal of documentation using different methodologies that confirms these views (from the large number of references here, we indicatively mention [5–13]).

The first problem that arises from EN 1998-1 is that the phenomenon of the torsional vibrations about the vertical axis, due to the developing inertia torsional moment (around Z-axis) of the building diaphragm, is ignored; this phenomenon occurs both in the linear and in the nonlinear area. In the linear area, and in the case of mono-symmetric single-storey buildings, this phenomenon has been fully investigated in the past [14–18] and the relevant research was completed with the extraction of precise closed mathematical relations [19]. Thus, the international scientific community ended up using "dynamic eccentricities", which is a fully documented proposal adopted by various seismic codes (N.B.C. Canada/1990 & 1995, DIN-4149, Draft Eurocode EC8/1989 & EC8/1998, Portugal/1986, Hellenic EAK/2000, etc.). It is clarified that dynamic eccentricities are added to the accidental eccentricity e_a, so that the lateral static floor force is always applied more eccentrically to the diaphragm concentrated mass. Also, with regard to the nonlinear area, according to Bosco et al. [20–22], an alternative new method of pushover analysis using dynamic eccentricities (called "corrective eccentricities) has been proposed, that refers back to the work of Makarios ([23–25]). All the above-mentioned articles are in contrast with EN 1998-1, since the latter Code does not use dynamic eccentricities.

The second problem, which also arises from EN 1998-1, refers to the fact that in the nonlinear area, where the pushover analysis is carried out, the issue of the orientation of the lateral static floor force remains completely open, since it is impossible to define the building's principal axes in the nonlinear area. This is an issue that has also concerned the international scientific community in the past [23,24,20,25].

Therefore, for the documented application of the pushover analysis, it is necessary to achieve the following threefold objective: (a) to define the origin point for the measurement of the appropriate dynamic eccentricities, (b) to calculate the magnitude of these dynamic eccentricities, and (c) to determine the appropriate horizontal orientation of the lateral static floor force. The solution to this triple problem is the subject of the present work that concerns the documented application of pushover analysis on asymmetric single-storey r/c buildings. The proposed pushover method of analysis aims directly at the Near Collapse (NC) limit state using the Displacement-Based concept.

2. Centre of rigidity vs centre of strength of asymmetric single-storey building

It is known that in r/c asymmetric single-storey buildings, the distance e_R (the Mass Centre CM from the Rigidity Centre CR of a building) is defined as the "static eccentricity" of the building. Here, we have to emphasize that there are several ways of calculating the Rigidity Centre CR, depending on the values of the moments of inertia of the member cross-sections to be used, as the position of the CR is different in the plan when we use (a) the moment of inertia I_{cr} of the uncracked cross-sections, with consideration to the reinforcement bars, that is suitable for use in the building's linear analysis for very small displacements (e.g. micro-vibrations of the buildings due to weak environmental causes), (b) the geometric moment of inertia I_g, which is related to the geometric cross-sections, that is suitable for use in the building's linear analysis for common small displacements, (c) the effective moment of inertia I_{ef} (reduced values) of the cross-sections, that is suitable for use with the linear model in the seismic design of new buildings using elastic design displacements (e.g. 50% reduction of the geometric moment of inertia I_g according to EN 1998-1), (d) the secant moment of inertia I_{sec} that leads to the secant (at yield) stiffness of the cross-section, that is suitable for use with the model in the nonlinear seismic analysis of the building, in order to estimate the direct inelastic design displacements (e.g. according to Eurocode EN1998-3). In each of the above cases, the position of the Rigidity Centre CR in the plan is different, but in the case of single-storey buildings, the CR always has the following three characteristic properties: (1) It is the Centre of Rigidity, because if the asymmetric single-storey building is loaded with any lateral static floor force at the CR-point, then the building diaphragm clearly moves in a translational manner, without twisting. (2) It is the Centre of Twist CT, because if the asymmetric single-storey building is loaded at the diaphragm level with an external floor moment (around the vertical Z-axis), then the building diaphragm rotates around the CT, which is the same point as the CR-point. (3) It is the Centre of Shear CS, because if the asymmetric single-storey building is loaded with any lateral static floor force (provided that the rotation of the building about the vertical axis is restrained) then the resultant of the internal column shear forces of all vertical members passes through this point, which is the same as the CR-point.

For the needs of the present paper and considering that the building-model must be examined in the nonlinear area in terms of deformation, inertia moments I_{sec} leading to the secant (at yield) stiffness of the cross-sections are used, i.e. the model proposed by EN 1998-3 for the seismic nonlinear analysis of buildings. In this case, the secant stiffness EI_{sec} is obtained as a constant value along the total length of each structural element and is equal to the numerical average of the EI_{sec} values of the two extreme element cross-sections, for positive and negative bending. The calculation of EI_{sec} (for the entire shear span length of any extreme element cross-section) is provided by a relationship included in informational annex A (Section A.3.2.4 (5)) of EN 1998-3 [26] as follows:

$$EI_{sec} = \frac{M_y}{\theta_y} \cdot \frac{L_v}{3} \tag{1}$$

The secant stiffness EI_{sec} depends on the yield moment M_y, the shear span length L_v and the chord rotation at flexural yield θ_y of the considered end cross-section of the member. Also, the chord rotation θ_y is determined by the contribution of purely flexural deformations at the end cross-section (as it is a function of the curvature φ_y at flexural yield,