Engineering Structures 166 (2018) 142–151

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

# Use of analytical lateral-axial strain relation in FE analysis of axially loaded rectangular CFST columns

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ARTICLE INFO

*Keywords:*
Concrete confinement
Concrete-filled steel tube (CFST)
Finite element analysis
Triaxial stress state

ABSTRACT

In a rectangular concrete-filled steel tube (CFST) column under axial compression, the concrete is under non-uniform and anisotropic confining stresses. To simulate such complicated triaxial stress field, finite element (FE) modelling is an effective tool. However, most previous FE models based on the plasticity theory augmented with associated or non-associated flow rules significantly underestimate the lateral and volumetric expansions of concrete at the inelastic stage and thus require solution dependent adjustments for compensation. Herein, an analytical lateral-axial strain relation of confined concrete derived from regression analysis of published experimental results is incorporated in a new FE model for the analysis of axial loaded rectangular CFST columns. This analytical lateral-axial strain relation allows direct evaluation of the lateral strains and should more accurately predict the lateral and volumetric expansions of confined concrete than the plasticity theory. The new FE model is applied to analyse tested specimens in the literature for verification and then used in a parametric study to evaluate the ductility of axially loaded rectangular CFST columns with different structural parameters.

## 1. Introduction

Since the 1960s, extensive experimental studies on concrete-filled steel tubes (CFST) have been carried out [1,2] and it is now well recognized that the inelastic lateral expansion of concrete under axial compression would induce lateral confining stresses acting onto the concrete to enhance its axial strength and ductility. For structural analysis and design, various theoretical models of CFST in the form of simple empirical models [3–5] or sophisticated finite element (FE) models [6–11] have been developed, among which the FE models are of course more powerful.

In most FE models, the plasticity theory is employed to model the constitutive behaviour of concrete and Drucker-Prager's (D-P) linear function [6–8] or hyperbolic function [9–11] is used to represent the plastic flow potential such that the increment of plastic volumetric strain $\varepsilon_v^p$ is equal to $\lambda_o \cdot \tan \psi_c$ where $\psi_c$ is the dilation angle normally set to be a constant and $\lambda_o$ is the stress dependent increment of effective plastic strain to be determined at each loading step to make sure the stress state lands on the D-P yield surface. However, such a constant $\psi_c$ setting is not able to capture the nonlinear lateral-axial strain relation of confined concrete. Yu et al. [12] revealed by comparing the FE analysis results based on plasticity theory with the experimental results that the then existing FE models significantly underestimate the lateral expansion of concrete under axial compression. Some novel plastic flow potential models have emerged [13,14], but they still underestimate the lateral and volumetric expansions of confined concrete at the inelastic stage [15].

Yu et al. [12] pointed out that, in order to calculate the confining stresses accurately, $\psi_c$ should be expressed as a function of plastic deformation, stress state and rate of confinement increment, or in other words, as a solution-dependent field variable (SDFV) in the language of the general-purpose FE software ABAQUS, thus necessitating an extra level of numerical iteration to achieve convergence. They derived the function for $\psi_c$ based on the assumption of $\sigma_1 = \sigma_2$, i.e. isotropic confining stresses, which is applicable only in certain limited cases, such as circular CFST under concentric compression. To enable the use of the SDFV approach when $\sigma_1 \neq \sigma_2$, they employed an effective confining stress calculated from $\sigma_1$ and $\sigma_2$ as a compromised solution [16]. Nevertheless, although the situation of $\sigma_1 \neq \sigma_2$ is fairly complicated, recent research demonstrated that the minimum of $\sigma_1$ and $\sigma_2$ can be treated as an effective confining stress for predicting the axial strain hardening and softening behaviour of confined concrete [17–19].

To overcome the above difficulties, it is proposed to directly evaluate the lateral strains of the confined concrete from the axial strain and confining stresses during the FE analysis. Frankly speaking, concrete is not exactly plastic and modification of the plasticity theory to improve its estimation of lateral strains is not theoretically sound. Herein, a method of incorporating an analytical lateral-axial strain relation of

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https://doi.org/10.1016/j.engstruct.2018.03.068
Received 6 November 2017; Received in revised form 11 February 2018; Accepted 23 March 2018

confined concrete in a new FE analysis framework for application to rectangular CFST columns is introduced. This FE analysis framework, implemented by a Fortran 90 program, is verified by analysing and checking against test results from the literature [20–23] and then employed in a comprehensive parametric study to evaluate the effects of aspect ratio, concrete cylinder strength, steel yield strength and steel tube thickness on the ductility of rectangular CFST columns.

## 2. FE modelling and numerical procedures

### 2.1. Modelling of concrete

The global coordinate system is denoted by "*x-y-z*", and the local coordinate system in the principal directions is represented by "1-2-3", where "*x-y*" and "1-2" are the lateral directions (i.e. the in-plane directions), and "*z*" and "3" stand for the axial direction (i.e. the normal-to-plane direction). The "*z*-axis" in the global coordinate system coincides with "3-axis" in the local coordinate system. Since the confined concrete to be analysed is mostly under compression, the sign convention adopted is "compression-positive".

By regression analysis of experimental results from many different sources, Dong et al. [24] have derived an analytical model for predicting the lateral strains of confined concrete. The principal lateral strain $\varepsilon_1$ in this model is expressed as:

$$\varepsilon_1 = -\nu_c\varepsilon_3 + (1-\nu_c^2)\frac{\sigma_1}{E_c} - \nu_c(1+\nu_c)\frac{\sigma_2}{E_c} + \varepsilon_1^p \tag{1a}$$

$$\varepsilon_1^p = -19.1(\varepsilon_3-\varepsilon_{3,1}^{lim})^{1.5}\left\{0.1 + 0.9\left[\exp\left(-5.3\left(\frac{\sigma_1}{f_c}\right)^{1.1}\right)\right]\right\} \tag{1b}$$

where $E_c$, $\nu_c$ and $f_c$ are the Young's modulus, Poisson's ratio and cylinder strength of the concrete. The term $\varepsilon_1^p$ in the above expressions is the inelastic lateral strain, which is equal to zero at the elastic stage and begins to take effect when splitting cracks are formed at $\varepsilon_3$ larger than the threshold strain $\varepsilon_{3,1}^{lim}$. The value of the threshold strain $\varepsilon_{3,1}^{lim}$ may be evaluated from:

$$\varepsilon_{3,1}^{lim} = \varepsilon_{co}(0.44 + 0.0021f_c - 0.00001f_c^2)\left[1 + 30\exp(-0.013f_c)\frac{\sigma_1}{f_c}\right] \tag{2}$$

in which $\varepsilon_{co}$ is the axial strain corresponding to the cylinder strength of the concrete. This axial strain may be taken as $\varepsilon_{co} = \frac{f_c}{E_c}\frac{4.26}{\sqrt[4]{f_c}}$ if there is no experimental data [25].

The strains $\varepsilon_2$, $\varepsilon_2^p$ and $\varepsilon_{3,2}^{lim}$ in the lateral principal direction 2 can be similarly obtained by substituting the subscript 1 in the above equations with the subscript 2. With the inelastic portions of $\varepsilon_1$ and $\varepsilon_2$, namely $\varepsilon_1^p$ and $\varepsilon_2^p$, so determined, the constitutive matrix equation of the concrete is derived as:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \lambda_c\begin{bmatrix}1-\nu_c & \nu_c & 0\\ \nu_c & 1-\nu_c & 0\\ 0 & 0 & \frac{1-2\nu_c}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_1-\varepsilon_1^p\\ \varepsilon_2-\varepsilon_2^p\\ \gamma_{12}\end{Bmatrix} + \lambda_c\nu_c\begin{Bmatrix}\varepsilon_3\\ \varepsilon_3\\ 0\end{Bmatrix} \tag{3a}$$

$$\lambda_c = \frac{E_c}{(1+\nu_c)(1-2\nu_c)} \tag{3b}$$

To model the axial strain hardening/softening behaviour of confined concrete under triaxial stress state, a triaxial failure surface is needed to defined the strength envelope of confined concrete when $\sigma_1 \neq \sigma_2$. Menétrey and Willam's failure surface [26] is opted for; it is given by the following mathematical expressions:

$$F(\xi,\rho,\theta) = \left(\sqrt{1.5}\frac{\rho}{f_c}\right)^2 + m\left[\frac{\rho}{\sqrt{6}\cdot f_c}r(\theta,e) + \frac{\xi}{\sqrt{3}\cdot f_c}\right] - c = 0 \tag{4a}$$

$$R(\theta,e) = \frac{4(1-e^2)\cos^2\theta + (2e-1)^2}{2(1-e^2)\cos\theta + (2e-1)[4(1-e^2)\cos^2\theta + 5e^2-4e]^{1/2}} \tag{4b}$$

$$m = 3\frac{(k\cdot f_c)^2 - f_t^2}{k\cdot f_c\cdot f_t}\cdot\frac{e}{e+1} \tag{4c}$$

where $\xi$, ρ and $\theta$ are the Haigh-Westergaard coordinates computed from the stress invariants $I_1$, $J_2$ and $J_3$. The parameter $e$ is the out-of-roundness parameter that determines the shape of intersection between the loading surface and the deviatoric plane and normally ranges from 0.5 (triangular shape) to 1.0 (circular shape). Assuming that the biaxial compressive strength of concrete $f_{bc}$ is equal to $1.5\cdot f_c^{0.925}$, as per Papanikolaou and Kappos [13], a conforming formula for $e$ can be derived as:

$$e = \frac{44.55\cdot f_c^{-0.075} + 6.75\cdot f_c^{-0.15} - 3}{89.1\cdot f_c^{-0.075} - 6.75\cdot f_c^{-0.15} + 3} \tag{5}$$

For a given concrete with certain cylinder strength, $e$ has a constant value.

Lastly, the axial stress-strain curve follows Attard and Setunge's model [25], which was derived from triaxial tests of concrete under active confinement. Its equations are given by:

$$\frac{\sigma_3}{f_{cc}} = \frac{a_1\left(\frac{\varepsilon_3}{\varepsilon_{cc}}\right) + a_2\left(\frac{\varepsilon_3}{\varepsilon_{cc}}\right)^2}{1 + a_3\left(\frac{\varepsilon_3}{\varepsilon_{cc}}\right) + a_4\left(\frac{\varepsilon_3}{\varepsilon_{cc}}\right)^2} \tag{6a}$$

$$\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + (17-0.06f_c)\left(\frac{f_r}{f_c}\right) \tag{6b}$$

where $f_{cc}$ is the confined strength and $\varepsilon_{cc}$ is axial strain corresponding to $f_{cc}$. The values of $f_{cc}$ and $\varepsilon_{cc}$ are determined by imposing the condition that the stress state ($\sigma_1$, $\sigma_2$, $f_{cc}$) lands on the failure surface and taking the value of $f_r$ as the minimum of $\sigma_1$ and $\sigma_2$ as mentioned before [17–19]. The coefficients $a_1$, $a_2$, $a_3$ and $a_4$ governing the shape of the stress-strain curve have different values for the ascending and descending branches. Their formulas can be found in Attard and Setunge's article [25] and are therefore not presented here again for brevity.

### 2.2. Modelling of steel tube

The steel is assumed to be linearly-elastic and perfectly-plastic. It is also assumed to yield according to the von-Mises yield surface defined by:

$$F_s = \sqrt{\frac{1}{2}[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6\tau_{xy}^2]} - \sigma_v = 0 \tag{7}$$

Since perfect plasticity means neither strain-hardening nor strain-softening at the post-yield stage, the von-Mises stress $\sigma_v$ is a constant equal to the yield strength of steel $f_y$. The associated flow rule is applied to calculate the increments of plastic strains $\varepsilon_x^p$, $\varepsilon_y^p$, $\varepsilon_z^p$ and $\gamma_{xy}^p$ in the steel:

$$\begin{Bmatrix}d\varepsilon_x^p\\ d\varepsilon_y^p\\ d\varepsilon_z^p\\ d\gamma_{xy}^p\end{Bmatrix} = \lambda_o\begin{Bmatrix}1.5(\sigma_1-\frac{\sigma_x+\sigma_y+\sigma_z}{3})/f_y\\ 1.5(\sigma_2-\frac{\sigma_x+\sigma_y+\sigma_z}{3})/f_y\\ 1.5(\sigma_3-\frac{\sigma_x+\sigma_y+\sigma_z}{3})/f_y\\ 3\tau_{xy}/f_y\end{Bmatrix} \tag{8}$$

where as mentioned before, $\lambda_o$ is the stress-dependent increment determined by a numerical process to keep the stress state in compliance with the yield surface.

The constitutive equation of the steel at element level is therefore given by:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\end{Bmatrix} = \lambda_s\begin{bmatrix}1-\nu_s & \nu_s & \nu_s & 0\\ \nu_s & 1-\nu_s & \nu_s & 0\\ \nu_s & \nu_s & 1-\nu_s & 0\\ 0 & 0 & 0 & \frac{1-2\nu_s}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x-\varepsilon_x^p\\ \varepsilon_y-\varepsilon_y^p\\ \varepsilon_z-\varepsilon_z^p\\ \gamma_{xy}-\gamma_{xy}^p\end{Bmatrix} \tag{9a}$$