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A novel tension estimation approach for elastic cables by elimination of complex boundary condition effects employing mode shape functions

Chien-Chou Chen, Wen-Hwa Wu*, Shin-Yi Chen, Gwolong Lai

Department of Construction Engineering, National Yunlin University of Science and Technology, Yunlin 640, Taiwan

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ABSTRACT

To establish a rigorous mathematical foundation for a recently developed tension determination method incorporating the mode shape functions, the frequency equations and mode shape functions for the vibrating cables with various types of boundary conditions are first derived in this research. According to these analytical expressions in terms of dimensionless parameters, it is confirmed that the mode shape functions associated with complicated boundary conditions are all dominantly contributed by the sinusoidal components to suggest a more convenient option than the intertwined frequency equations. Based on this discovery, the cable tension can be generally decided with an explicit formula similar to that for the case with hinged boundary constraints at both ends. The effective vibration length for each selected mode in the formula, however, needs to be pre-determined by fitting the corresponding sinusoidal function with the mode shape ratios identified from multiple synchronized measurements. Parametric study is also conducted on the derived mode shapes to systematically investigate the interference effect of the hyperbolic component near the boundaries and provide valuable guidelines for the sensor deployment in engineering applications. Finally, the performance of the developed methodology in the cases with more involved boundary constraints is certified with demonstrative numerical examples and laboratory experiments.

1. Introduction

Tension determination typically plays the most crucial role in the structural health monitoring of cable-stayed bridges with stay cables [1,2], box girder bridges with external tendons [3,4], and through-type arch bridge with suspenders [5,6]. Even though a number of devices such as hydraulic jacks, strain gauges, load cells, embedded fiber Bragg grating (FBG) sensors, and elasto-magnetic (EM) sensors have been adopted to evaluate the tension of these critical force-transmitting members, problems like doubtful accuracy, high cost, complicated installation, or poor endurance are regularly encountered in practical applications. Taking advantage of the relatively simple requirements in analysis and measurement due to one-dimensional geometry, the ambient vibration method is more popularly utilized in the practical tension estimation for a stay cable, external tendon, or suspender [1,2,7,8].

The vibration-based tension estimation approach usually starts with identifying the modal frequencies from the ambient vibration measurements and then employing a pre-determined formula or finite element (FE) analysis to decide the tension. According to the string theory based on the simplified model of a transversely vibrating string with hinged boundary conditions at both ends, the tension of an elastic cable or tendon is conventionally obtained from an analytical formula requiring the identified frequencies together with the given vibration length and mass per unit length. The accuracy of this ambient vibration method can be further improved by developing more advanced formulas in an analytical or empirical sense to include the effects of flexural rigidity [9–12], gravity sag [13–15], and complicated boundary constraints [11–14,16–20]. In addition, FE analysis [21–24] or finite difference method [25] has been alternatively applied in a few studies to search for the optimal values of tension, flexural rigidity, and other parameters that would best fit the identified modal frequencies. The current development for this type of methodology is focusing on the application of non-contact sensors [26–30] and the investigation of time-varying cable tension [31,32]. These efforts, however, may not be as important as the appropriate selection of vibration length to more faithfully reflect the real vibration behavior. In practical situations, special anchorage systems with rubber constraints are normally installed near the boundaries of a stay cable or suspender. External dampers may also be mounted on long stay cables near the deck end to alleviate wind-induced vibrations. Besides, intermediate diaphragms often appear in the application of external tendons to adjust the tendon direction. Such design details definitely result in much more involved

* Corresponding author.
E-mail addresses: ccchen@yuntech.edu.tw (C.-C. Chen), wuwh@yuntech.edu.tw (W.-H. Wu).

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boundary conditions and raise a great challenge in deciding an accurate vibration length.

For efficiently handling the difficulties associated with complicated boundary constraints, an innovative concept combining the mode shape functions and the modal frequencies was recently proposed by the authors to develop an accurate tension determination method [33–37]. Multiple synchronized vibration measurements were first taken and identified to determine the mode shape ratios at various sensor locations for each observable mode. Based on these ratios, a pivotal procedure was then conducted to search the best fitted sinusoidal mode shapes for independently obtaining the effective vibration length of each considered mode. With such lengths attained for several selected modes, the tension and flexural rigidity can be subsequently solved by simple linear regression techniques using the identified modal frequencies. This methodology was initially simplified by assuming symmetric boundary constraints [33,34] which are usually adopted in most of the practical anchorage systems for stay cables. Generalization has also been made with the introduction an additional shifting parameter in the assumed mode shape function to effectively consider the unsymmetrical boundary constraints [35,36]. It was further discovered that such an unsymmetrical formulation would result in an intricate objective function difficult to obtain the optimal vibration length if all the sensors are concentrated at one end. Nevertheless, this problem can be conveniently tackled by adding one sensor near the other end for the case of external tendon [35,36]. As for the cases where the far end is not conveniently accessible, a methodology was developed in a most recent work [37] to deal with the suspenders of arch bridge. Making use of the fact that the hinged or fixed connection is frequently designed at the arch end [38] in such applications, the optimal shifting parameter was employed to decide the virtual hinged boundary for a feasible tension determination simply with multiple measurements concentrated at the deck end. Other researchers have also begun delving into similar ideas in the latest few years [39–43].

Although the new method incorporating the sinusoidal mode shape functions has been successfully applied in several practical cases including stay cables [33–36], external tendons [35,36], and suspenders [37], the legitimacy of using an equivalent cable with hinged constraints at both ends to represent the actual case with complicated boundary constraints may still be questioned. In these studies [33–37], the whole methodology is essentially built on a qualitative explanation that the effect of anchorage device should be limited in a local range and the primary free length section in the middle of cable ought to be eligibly modeled by the equivalent cable. It is apparently not adequate. A recent work by Yan et al. [42] made an attempt by considering rotational springs at both ends to more systematically quantify this concept. Following such a development, the current paper is aimed to establish a rigorous mathematical foundation for this methodology with more solid derivations and comprehensive analysis. Moreover, it has been indicated [37] that the accuracy in tension estimation can be deteriorated if the sensors are installed at the positions extremely close to the boundaries. Parametric study on the analytical solutions derived in this work is also carried out to demonstrate that more specific guidelines of sensor deployment can be suggested for different types of pre-stressed members. Finally, the numerical verification and laboratory validation with an experimental strand are further conducted to more systematically assess this promising approach.

2. Solutions for cable vibration equation in terms of dimensionless parameters

In this section, the cable vibration equation is expressed with dimensionless parameters and its solutions are analytically derived in a general sense. Apart from the hinged [9,10], clamped [11,42], and rotationally elastic boundary conditions [11,42] already investigated in the literature, more complicated cases are also considered herein for a thorough examination on the boundary effects in the subsequent sections. It needs to point out that the effect of sag-extensibility is neglected in the current work for the viability of analytical derivations. In fact, the progress in modern stay cable production to enhance the steel strength, reduce the weight per unit length, and replace cement grouting by lighter materials also diminishes the importance of considering this issue. According to the practically identified cable frequencies from real measurements [34,36], the sag effect may be only observable in the fundamental mode for most stay cables or in the first few modes for extremely long cables. Since numerous modal frequencies of cable are typically identifiable in engineering applications, the complexity of sag consideration can be dodged in the vibration-based tension determination approach by avoiding the fundamental frequency or the first few modal frequencies. The other effective recipe is to employ the out-of-plane cable vibration measurements.

2.1. Equation of motion and general solution

The transverse displacement $v(x, t)$ of a cable subjected to an axial tension T and under free vibration is a function of axial coordinate x and time t to be governed by the following equation of motion:

$$EI\frac{\partial^4 v}{\partial x^4} - T\frac{\partial^2 v}{\partial x^2} + \overline{m}\frac{\partial^2 v}{\partial t^2} = 0 \tag{1}$$

where $\overline{m}$ signifies the mass per unit length, E stands for the Young's modulus, and I denotes the cross-sectional area moment of inertia. Further adopting the cable length L to define a dimensionless variable $\overline{x} = x/L$ in a range of $0 \leqslant \overline{x} \leqslant 1$, Eq. (1) can be transformed into:

$$\frac{EI}{L^4}\frac{\partial^4 v}{\partial \overline{x}^4} - \frac{T}{L^2}\frac{\partial^2 v}{\partial \overline{x}^2} + \overline{m}\frac{\partial^2 v}{\partial t^2} = 0 \text{ or } \frac{\partial^4 v}{\partial \overline{x}^4} - \varepsilon\frac{\partial^2 v}{\partial \overline{x}^2} + \beta^2\frac{\partial^2 v}{\partial t^2} = 0 \tag{2}$$

where

$$\varepsilon \equiv \frac{TL^2}{EI} \text{ and } \beta \equiv \sqrt{\frac{\overline{m}L^4}{EI}} \tag{3}$$

are both positive parameters. For solving Eq. (2), separation of variables is typically applied by letting $v(\overline{x},t) = \phi(\overline{x})Y(t)$ to yield

$$\begin{cases} \phi^{iv}(\overline{x}) - \varepsilon\phi''(\overline{x}) - \beta^2\omega^2\phi(\overline{x}) = 0 & \text{(a)} \\ \ddot{Y}(t) + \omega^2 Y(t) = 0 & \text{(b)} \end{cases} \tag{4}$$

where ω^2 is an arbitrary constant. Based on Eqs. (4a) and (4b), the general solutions of $\phi(\overline{x})$ and $Y(t)$ can be conveniently determined [9] as:

$$\phi(\overline{x}) = A_1\sin\delta\overline{x} + A_2\cos\delta\overline{x} + A_3\sinh\gamma\overline{x} + A_4\cosh\gamma\overline{x} \tag{5}$$

and

$$Y(t) = B_1\sin\omega t + B_2\cos\omega t \tag{6}$$

where

$$\delta \equiv \sqrt{\sqrt{\beta^2\omega^2 + \frac{\varepsilon^2}{4}} - \frac{\varepsilon}{2}} \text{ and } \gamma \equiv \sqrt{\sqrt{\beta^2\omega^2 + \frac{\varepsilon^2}{4}} + \frac{\varepsilon}{2}} \tag{7}$$

are both positive parameters. From Eq. (6), it should be noted that ω actually represents the natural frequency of the cable in rad/sec and $\beta\omega$ can be regarded as a normalized frequency without dimension. In addition, ε, δ and γ are also dimensionless. For the convenience in the following derivations, Eq. (5) is further reformulated as:

$$\phi(\overline{x}) = C_1\sin\delta\overline{x} + C_2\sin\delta(1-\overline{x}) + C_3\sinh\gamma\overline{x} + C_4\sinh\gamma(1-\overline{x}) \tag{8}$$