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A punching shear mechanical model for reinforced concrete flat slabs with and without shear reinforcement

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ABSTRACT

A punching shear strength mechanical model for RC flat slabs with and without shear reinforcement, based on a beam shear model previously developed by the authors, is presented. The differences in resisting actions between beam shear and punching shear have been identified and incorporated into the governing equations and failure modes, resulting in simple but accurate punching design equations. The model consistently explains and quantifies some experimentally observed phenomena, such as the higher contribution of the concrete to the punching shear strength of slabs than in the case of beams subjected to shear, mainly due to the multi-axial state of stresses that takes place near the support. Furthermore, the model provides physical meaning to some parameters used in the design, such as the position of the critical perimeter or the effective stress of the punching reinforcement, among others. Very good agreement has been obtained between the model predictions and the results of 560 punching tests of concentrically loaded slabs, with and without shear reinforcement, included in two available large databases. The mechanical character of the model allows its extension to post-tensioned flat slabs, border or corner columns, steel fiber and FRP reinforced concrete slabs or different strengthening systems in a consistent way.

1. Introduction

Punching capacity of slabs has been extensively studied in the past, both from the experimental and theoretical viewpoints [1–29]. As a result of these research works, several approaches have been developed for predicting the punching strength of reinforced concrete slabs with and without shear reinforcement. Even though some developed models reproduce quite well the experimental results, there is not yet a generally accepted design model which combines accuracy with the necessary simplicity for daily design, adaptable to the variety of situations that can take place in practice. This is evidenced by the differences in the treatment of the punching strength in relevant codes provisions, such as EC2 [30], ACI [31] and Model Code 2010 [32], or by the changes produced along the time in some essential design parameters. Some examples of aspects still in discussion are the position of the critical perimeter, the effective stress in the shear reinforcement at ULS or the influence of the presence of shear reinforcement on the punching concrete contribution, among others. In fact, many of the punching strength code provisions are based on empirical models, adjusted to tests results, but without a consistent theory behind.

Certainly, advanced numerical models are more and more capable to simulate the local and global observed punching behavior [33–37]. However, there is still the need to improve the objectivity of the models, which are excessively dependent on the materials parameters used (i.e., post-cracking and softening behavior, bond…) in order to obtain reliable predictions of the experimental results without requiring too much effort and time. Nevertheless, numerical methods have become very useful tools to provide support to the development of conceptual models, by allowing the verification of certain assumptions and quantifying the influence of certain variables by performing parametric studies.

Since punching shear is a brittle -and therefore undesirable- failure, in order to reach the required safety level without an unaffordable cost, simplified, but safe and accurate design models are needed. In order to achieve these characteristics, such models should be based on the principles of concrete mechanics and should be verified with available experimental results.

In this paper, a new mechanical model for the estimation of the punching shear strength of reinforced concrete flat slabs with and without shear reinforcement is presented. The punching shear model presented in this paper is an adaption of a previously existing model for beam shear strength, developed by the authors in [38–40], which

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Nomenclature

Notations

a	shear span. For slab floors in buildings subjected to distributed loads, the shear span, a, to be used in the size effect parameter, ζ, can be estimated as the average distance from the position of the line of zero radial bending moment to the edge of the column, $l_0 = \sqrt{l_{0y} \cdot l_{0z}}$, where $l_{oy} \cong 0.15\ l_y$ and $l_{oz} \cong 0.15\ l_z$, l_y and l_z are the span lengths in the y and z directions. See Ref. [40] for complete definition regarding shear in beams
b	width of the cross-section of a beam. For T or I-shaped is equal to the flexural effective compression flange width
b_w	width of the web on T, I or L beams. For rectangular beams $b_w = b$
d	effective depth of the cross-section
d_0	effective depth of the cross-section, d, but not less than 100 mm
d_s	distance between the maximum compressed concrete fibre and the centroid of the mild steel tensile reinforcement. In the case of prestressed elements without mild reinforcement, d_s shall be taken equal to d_p
d_p	distance between the maximum compressed concrete fibre and the mechanical centroid of the prestressing tendons placed at the tension zone
f_{cc}	confined concrete compressive strength
f_{cd}	design value of concrete compressive strength
f_{ck}	characteristic compressive strength of concrete
f_{cm}	mean compressive strength of concrete
f_{ctm}	mean tensile strength of concrete, equal to $0.30 \cdot f_{ck}{}^{2/3}$ in MPa, not greater than 4.60 MPa
f_{yw}	mean yield strength of the shear reinforcement
f_{ywd}	design yield strength of the shear reinforcement
m_{crack}	slab cracking bending moment per unit width
m_r	bending moment per unit length producing radial stresses around the column
m_φ	bending moment per unit length producing tangential stresses around the column
r	radial distance from the column axis
r_0	radial distance from the column axis to the point of zero radial bending moment (contraflexure point)
r_{col}	radius of a column with equal perimeter than the actual column
r_{crack}	distance from the starting point of the critical crack (due to bending) to the column axis, see Fig. 7
r_{crit}	distance from the critical perimeter to the column axis, see Fig. 7
s	radial distance
s_{crack}	distance from the starting point of the critical crack (due to bending) to the column face, see Fig. 7
s_{crit}	distance from the critical perimeter to the column face, $s_{crit} = 0.5d$, see Fig. 7
u_{crit}	critical perimeter (Fig. 7) placed at a distance s_{crit} from the column face
u_{out}	perimeter where shear reinforcement is not longer required (Fig. 10) (see Fig. 2).
x	neutral axis depth of the cracked section, obtained assuming zero concrete tensile strength
x_0	neutral axis depth of a RC member or of a PC member considering P = 0 and the same amounts of reinforcements
z	inner lever arm. In the shear analysis of reinforced concrete beams without axial force, the approximate value $z \approx 0.9d$ may normally be used. See Eq. (2)
A_{sw}	cross-sectional area of the shear reinforcement. For punching, Asw is the total area of the shear reinforcement placed around the column that crosses the critical inclined crack that can be approximated by considering the reinforcement placed between 0.5d and 1.5d from the support face
E_{cm}	secant modulus of elasticity of concrete, $E_{cm} = 22000(f_{cm}/10)^{0.3} \ngtr 39$ GPa
E_s	modulus of elasticity of reinforcing steel
G_f	concrete fracture energy, $G_f = 0.028 f_{cm}^{0.18} d_{max}^{0.32}$, in Eq. (4)
K_c	factor equal to the relative neutral axis depth, x/d, but not greater than 0.20, in Eqs. (10) and (31)
K_s	factor that accounts for the effectiveness of the anchorage of the shear reinforcement
M_{cr}	cracking moment at the section where shear strength is checked calculated using the mechanical properties of the gross concrete section and the flexural tensile strength
M_{Ed}	concomitant design bending moment, considered positive
V_{Ed}	design shear force in the section considered
V_{Rd}	design shear resistance of the member
$V_{Rd,max}$	design value of the maximum shear force which can be sustained by the member, limited by crushing of the struts
V_u	shear resistance of the member calculated by the background mechanical model, Eq. (1)
$V_{u,max}$	maximum shear force which can be sustained by the member, limited by crushing of the struts in the background mechanical model or multi-action model, Eq. (2)
α	angle between shear reinforcement and the beam axis perpendicular to the shear force in Eqs. (11) and (32). In Eq. (18) α is a parameter taking into account the non-uniform distribution of the vertical stresses
α_{cw}	coefficient taking account the state of the stress in the struts. See EC2 [30] for further information.
α_e	modular ratio, $\alpha_e = E_s/E_{cm}$
α_{max}	parameter for the determination of the maximum punching shear capacity, Eq. (35)
ν	Poisson coefficient
ν_1	strength reduction factor for concrete cracked in shear. See EC2 [30] for further information
θ	angle between the concrete compression strut and the beam axis perpendicular to the shear force
ρ_l	longitudinal tensile reinforcement ratio. The neutral axis depth x/d should be obtained using the average of the longitudinal reinforcement ratios, ρ_{lx}, ρ_{ly}, in the two orthogonal directions, adopting an effective slab width $b_{s,eff}$ approximately equal to the column side or diameter plus 3 times the slab effective depth at each side of the column
σ_r	normal radial stresses around the column produced by m_r
σ_φ	normal tangential stresses around the column produced by m_φ
σ_v	vertical stresses in the slab in the vicinity of the column, see Fig. 5
ζ	size and slenderness effect factor, given by Eq. (12)

incorporates the contribution of the main shear resisting mechanisms. For this purpose, the relevant differences between the shear in beams and punching shear resisting actions have been identified and accounted for into the governing equations and into the failure criteria used in the mechanical model. Numerical simulations using a non-linear finite element model have been used to verify some of the assumptions made. Different authors have developed very complete and comprehensive databases on punching tests performed on