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Lateral buckling of tapered thin walled bi-symmetric beams under combined axial and bending loads with shear deformations allowed

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ABSTRACT

In this paper, the effect of shear deformations on elastic lateral buckling of tapered thin-walled open and closed bi-symmetric section beams under combined bending and axial forces is investigated. For the purpose, a geometrical non-linear beam theory is presented according to a new kinematic model that incorporates shear flexibility components. Ritz method is applied to obtain the governing equilibrium equations, then the buckling loads are computed by solving the eigenvalue problem basing on the singularity of the tangential stiffness matrix. The elastic lateral buckling resistances given by the proposed method are generally in good agreement with the finite element simulation using Abaqus software. The numerical results for cantilevers and simply supported thin walled beams with open and box sections reveal that the classical stability solution tends to overestimate the real lateral buckling resistance of short tapered box cantilever beams.

1. Introduction

Structural elements with uniform sections that are made of steel and designed for their structural efficiencies are widely employed in frames, although the required resistance may be considerably lower.

Tapered elements seem a good choice to substitute standard prismatic elements since they lead to material savings and reduce environmental impact. Therefore, a better carbon trace of steel industry is achieved.

However, to take the most advantage of this, the designer must take in the account the instability phenomenon of thin-walled elements.

It's admitted that a great majority of laterally unrestrained beams and cantilevers either prismatic or tapered, present lateral torsional buckling phenomena.

Thin walled beams flexural-torsion behavior is complex and predominated by warping and bending torsion coupling [1,2].

Tapered thin-walled elements buckling behavior is rather a more difficult task compared to those with uniform sections because the tapered elements geometrical parameters are not constant and vary along the axial position.

From analytical point of view, equilibrium equations are increasingly more complicated to apprehend with the presence of variable stiffness coefficients and also very hard to get an easy practical solution. Therefore, it is not surprising that in the design codes such as European code "Eurocode 3" [3], no recommendations are addressed for the case of tapered beams strength. It is noteworthy that, the accurate prediction of the stability limit state is of fundamental importance while designing this kind of structural elements. Early studies on tapered elements under torsion loading have been carried by Brown [4]. The author proposed a theoretical approach of thin-walled tapered beams behavior with additional terms.

Bradford and Cuk [5] used the finite element method and proposed a general approach to study the torsion bending (combining thin walled beams and open variable cross sections).

Wekzer [6,7] presented an interesting approach based on the shell theory in order to establish the equilibrium equations of tapered thin-welled beams with open sections.

Yeong and Jong [8] have provided the solution for the instability problem of tapered thin-walled elements by the finite element method through the establishment of linear and geometric stiffness matrices.

Chan [9] investigated a kinematical model based on the definition of the nonlinear axial deformation which was identical to a prismatic beam case.

Rajasekaran [10] developed a nonlinear kinematic model for thin walled tapered beams with open sections by using the cosines director matrix of Love [11]. Several studies on lateral buckling of tapered thin walled elements have been undertaken by using the linear approach, Vlasov [12].

Ronagh et al. [13] used the finite element method in the case of moderate torsion amplitudes of thin walled tapered beams with doubly symmetric cross sections.

In the same way, according to the variational formulation, Andrade

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et al. [14,15] analyzed the elastic lateral-torsional buckling behavior of singly symmetric thin-walled tapered beams by Ritz's method.

Lei and Shu [16] presented a new theoretical model for the lateral buckling of web tapered I beams, according to the thin-walled Vlasov's assumptions.

Asgarian et al. [17] and Soltani et al. [18] used the power series method, both in the bending stability and vibration analyses of tapered thin-walled beams.

Serma et al. [19] presented an approximate fast methodology that can be applied to columns with a general stiffness variation and subjected to any axial loading distribution.

More recently, Trahair [20] described a simple finite element model for lateral buckling resistance of tapered beams and cantilevers which is based on the classical linear stability theory.

The recent contribution, in this field, has been addressed by Benyamina et al. [21]. The authors presented a new kinematic model for the web tapered I beam sections. In this work, new geometrical parameters are used to describe correctly the lateral torsion buckling behavior of the structure.

Mohri et al. [22] have formulated a 3D finite element beam for a nonlinear buckling analysis of tapered thin walled cantilevers in the large torsion behavior.

It is well known, that the classical beam theories, which neglected, transverse shear deformation have often failed to predict the behavior of closed section beams in lateral buckling resistance. Within this context, many investigations have been dedicated to the effects of shear deformations on the behavior of composite thin walled structures under bending and torsion loads. Pluzsik et al. [23] have proposed a novel analytical model of simply supported composite beams with closed sections subjected to sinusoidal loads. In their model the effects of restrained warping and shear deformation were taken into account. In the same way Laszla et al. [24] have employed the same model for the flexural torsional buckling of composite beams with open sections. Sebastian et al. [25] have shown that the classical theory that omits the shear flexibility is inaccurate for the lateral buckling prediction. In recent work Ziane et al. [26] developed an exact dynamic stiffness matrix to investigate the free vibration of FGM box beams.

In spite of the large amount of work devoted to the analysis of the shear deformation effects on the behavior of composite thin-walled beams, the investigation of this effect on tapered thin-walled elements seems rather scarce.

The main objective of the present work is to investigate the shear deformation effect on the elastic buckling resistance, either for open and closed bisymmetric sections tapered thin-walled beams subjected to combined bending and axial forces.

For this purpose, the displacement and deformation relationships are first exposed without any simplifying assumption on the twist angle amplitude, in the presence of the shear flexibility terms.

In this investigation, the elastic lateral buckling equilibrium equations of tapered thin walled elements with bisymmetric cross-section under combined bending and axial forces are derived, being based on Ritz's method. The resulting equations are nonlinear and coupled.

Secondly, the buckling loads are determined from the singularity condition of the tangential stiffness matrix, evaluatedat the fundamental state. Many applications which consist on open and box sections are considered for tapered thin-walled beams and cantilevers. The obtained results are compared with finite element solution using Abaqus software [27]. The limits of the classical stability solution that ignores the shear deformation flexibility is discussed in this study.

2. Theoretical model for the stability analysis

2.1. Kinematics

A straight tapered thin walled beam with bi-symmetrical section is pictured in Fig. 1. The points within the structure is refered to a

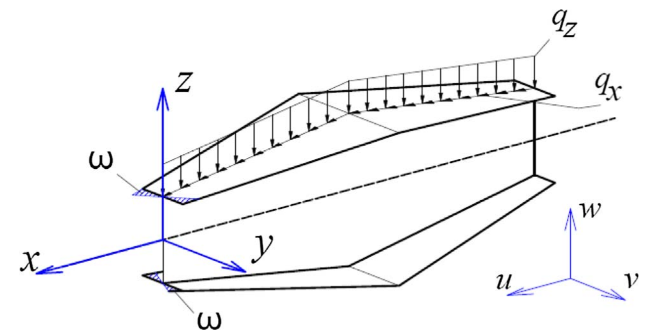

Fig. 1. Thin-walled beam, view of reference axes, displacement components and loads.

cartesian co-ordinate system (x, y, z). Let us denote by x, the longitudinal axis and y and z as the first and second principal bending axes. The origin of these axes is located at the centroid G.

Consider ***M***, a point on the section contour with its co-ordinates (y, z, ω). ω is the sectorial co-ordinate of the point used in Vlasov's model [12]. The fundamental assumptions of the theory of thin-walled elements are:

1. Linear elastic and homogeneous material.
2. Unchanged-contour hypothesis, the contour of thin wall does not deform in its own plane
3. Shell force and moment resultants corresponding to the circumferential stress σ_{ss} a γ_{ns} are neglected.

From these assumptions, the approximation of the displacements of a point ***M*** can be derived from those of the shear center ***C*** to be in the following form:

$$u_M = u - y(\phi_z \cos\theta_x + \phi_y \sin\theta_x) - z(\phi_y \cos\theta_x - \phi_z \sin\theta_x) - \omega\theta'_x \quad (1)$$

$$v_M = v - z\sin\theta_x - y(1-\cos\theta_x) \quad (2)$$

$$w_M = w + y\sin\theta_x - z(1-\cos\theta_x) \quad (3)$$

In this formulation, u is the axial displacement of centroid G. ϕ_y and ϕ_z are measures of the rotations about the shear center axis y and z respectively. For the thin walled beams with bi-symmetric sections, the shear center C **coincides** with G.v and w are displacement components of shear center in y and z directions and θ_x is the torsion angle.

In these equations, (.)′ denotes the first x-derivative. This expression is a generalization of others previously proposed in the literature. The displacement field proposed by Mohri et al. [2] is verified by considering $\phi_y = w'$ and $\phi_z = v'$. An alternative description of the displacement field is considered according to the tapered beam segment (tt') as shown in Fig. 2. In this Figure, the axial displacement along the segment (tt') denoted by $u_M^{tt'}$ is given by:

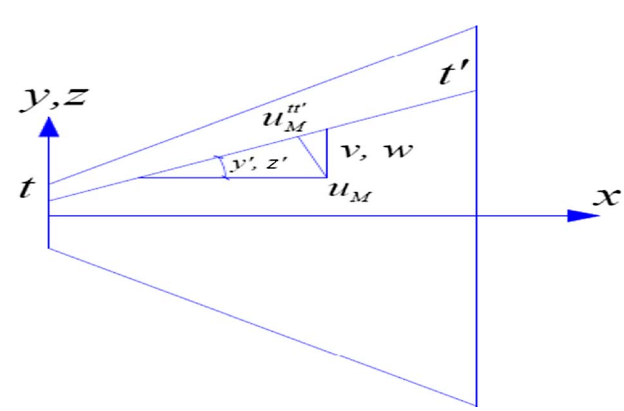

Fig. 2. Axial displacement according to the tapered beam segments.