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Dynamic analysis of functionally graded porous structures through finite element analysis

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ABSTRACT

A finite element method (FEM) analysis framework is introduced for the free and forced vibration analyses of functionally graded porous (FGP) beam type structures. Within the proposed computational scheme, both Euler-Bernoulli and Timoshenko beam theories have been adopted such that the explicit stiffness and mass matrices for 2-D FGP beam element through both beam theories are explicitly expressed. Both Young's modulus and material density of the FGP beam element are simultaneously considered as grading through the thickness of the beam. The material constitutive law of a FGP beam is governed by the typical open-cell metal foam. Furthermore, the damping effects of the FGP structures can be also incorporated within the proposed FEM analysis framework through the Rayleigh damping model. Consequently, the proposed approach establishes a more unified analysis framework which can investigate simple FGP beams as well as complex FGP structural systems involving mixture of different materials. In order to demonstrate the applicability, accuracy, as well as the efficiency of the proposed computational scheme, both FGP beams and frame structures with multiple porosities have been rigorously explored.

1. Introduction

From the continuous development of modern society, there has always been a high demand for new building materials for various innovative engineering structures. As a general trend, high cost-effective materials are more desirable than ever before, so safer, cheaper, and more importantly, greener engineering projects can be anticipated [1–3].

Among numerous ingeniously invented engineering materials, the functionally graded porous material (FGPM) has become one of the research foci across many engineering disciplines [4–6]. As one of the advanced new generation composite materials, the FGPM has inherited the advantages from both traditional functionally graded materials and metallic porous materials by having attractive characteristics such as superb stiffness-to-weight ratio, energy dissipation, mechanical damping, as well as designable vibrational frequency [7]. Examples of the implementation of FGPMs in engineering applications are including sandwich panels with steel foam cores, balcony platforms, parking floor slabs, and race car crash absorbers, etc. [7]. Consequently, the FGPM has offered an opportunity for next-generation structures (i.e., at both macro-and micro-scales) possessing robust level of safety but with lighter weight and less carbon footage.

Since the emerging of such engineered composite material, the research work on FGPM has attracted numerous global research attentions. In 1964, Biot [8] proposed a poroelasticity theory for buckling analysis of fluid-saturated porous slab through the framework of thermodynamics of irreversible processes. Magnucki and Stasiewicz [9] investigated the elastic buckling problem for porous beams subjected to static compressive forces through an energy approach. Magnucka-Blandzi [10] studied the dynamic buckling problem for a metal foam circular plate through an analytical approach by assuming the middle plane is being symmetric. Jasion et al. [11], assessed the static stability of sandwich plates with metal foam cores. Both global and local buckling issues of the sandwich plates were investigated through the analytical and numerical approaches, and the results on the critical buckling loads were also compared with experiments for plates with various boundary conditions. Moreover, the buckling analysis of cylindrical shells made of porous materials subjected to the combination of axial loads and external pressure was investigated by Belica and Magnucki [12]. The geometric nonlinearity was also incorporated and the static critical buckling load was determined through an analytical analysis framework. Jabbari et al. [13] proposed a buckling analysis for

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porous circular plate with piezoelectric layers. Subsequently, a new analytical approach was proposed in [14] for the static buckling analysis of porous soft ferromagnetic FG circular plates under the influence of magnetic fields.

In addition to the buckling analysis of plates made of various porous materials, Chen et al. [15–17] investigated the problem of static bending, elastic buckling, free vibration, forced vibration, as well as nonlinear free vibration for FGP beams with the consideration of shear effects through an analytical analysis framework. Furthermore, Kitipornchai et al. [18] proposed an analytical approach for free vibration and static buckling analyses for FGP beams with nano-graphene platelet reinforcements. Soon after that, Chen et al. [19] extended their analytical approach to investigate the problems of nonlinear vibration and postbuckling for graphene reinforced FGP beams.

Even though there are some studies have been reported on various structural analyses for FGP beams, it is noticed that all aforementioned works regarding the dynamic analysis (free and forced vibration) of FGP beams have been proposed through the analytical analysis framework. Moreover, another common feature that can be realized from the previous studies is that only FGP beams with various boundary conditions subjected to different loading regimes have been investigated. However, for real-life engineering applications, complex structural systems (e.g., frames and trusses) are often being implemented to serve various engineering purposes. Therefore, it is essential to develop other types of analysis framework which should have the same level of accuracy as the analytical approach does, but with more extensive applicability.

In this study, a finite element method (FEM) analysis framework is proposed for free and forced vibration analyses of FGP beam type structures. Within the proposed computational scheme, both Euler-Bernoulli and Timoshenko beam theories have been adopted such that the explicit stiffness and mass matrices for 2-D FGP beam element through both beam theories are explicitly expressed. In addition to the capability on performing valid dynamic analysis for FGP beams, the proposed approach is also applicable to FGP structural systems (e.g., frames and trusses) with multiple types of functionally graded materials. Also, the damping effects of the FGP structural systems have been incorporated within the proposed computational scheme through the implementation of the Rayleigh damping model. Consequently, a more unified analysis framework is proposed in this study which rigorously maintains the accuracy level of analytical approaches, but at same time possesses the freedom of solving more complex structural systems involving diversity of engineered composite materials.

In order to achieve a more effective presentation, this paper is organized as follows. Section 2 introduces the proposed FEM for static analysis of FGP frame structures. Subsequently, the dynamic analysis of FGP frame structures through FEM is presented in Section 3. In order to demonstrate the applicability, accuracy and efficiency of the proposed computational scheme, three distinctive numerical examples have been thoroughly investigated in Section 4. Finally, the conclusion of this investigation is drawn in Section 5.

2. Static analysis of FGP frame structures through FEM

2.1. Material models for FGPM

Within the scope of this study, it is presumed that the material properties (i.e., Young's modulus, shear modulus, and material density) of a generic FGP beam element with rectangular cross section are continuously varying along the beam thickness direction. In order to achieve a more generalized analysis framework, two distinctive constitutive laws of the FGPM are considered which are including the symmetric and monotonic material constitutive relationships (i.e., SMCR and MMCR). Specifically, the SMCR model of the ith FGP beam element can be expressed as [15–17]:

$$\text{SMCR}\begin{cases} E^i(z) = E_1^i\left[1-\cos\left(\frac{\pi z}{h^i}\right)\right] + E_0^i\cos\left(\frac{\pi z}{h^i}\right) = E_1^i\left[1-e_0^i\cos\left(\frac{\pi z}{h^i}\right)\right] \\ G^i(z) = G_1^i\left[1-\cos\left(\frac{\pi z}{h^i}\right)\right] + G_0^i\cos\left(\frac{\pi z}{h^i}\right) = G_1^i\left[1-e_0^i\cos\left(\frac{\pi z}{h^i}\right)\right] \\ \rho^i(z) = \rho_1^i\left[1-\cos\left(\frac{\pi z}{h^i}\right)\right] + \rho_0^i\cos\left(\frac{\pi z}{h^i}\right) = \rho_1^i\left[1-\rho_m^i\cos\left(\frac{\pi z}{h^i}\right)\right] \end{cases} \tag{1}$$

and the MMCR model of the ith FGP beam element can be expressed as [15–17]:

$$\text{MMCR}\begin{cases} E^i(z) = E_1^i\left[1-\cos\left(\frac{\pi z}{2h^i}+\frac{\pi}{4}\right)\right] + E_0^i\cos\left(\frac{\pi z}{2h^i}+\frac{\pi}{4}\right) \\ \quad = E_1^i\left[1-e_0^i\cos\left(\frac{\pi z}{2h^i}+\frac{\pi}{4}\right)\right] \\ G^i(z) = G_1^i\left[1-\cos\left(\frac{\pi z}{2h^i}+\frac{\pi}{4}\right)\right] + G_0^i\cos\left(\frac{\pi z}{2h^i}+\frac{\pi}{4}\right) \\ \quad = G_1^i\left[1-e_0^i\cos\left(\frac{\pi z}{2h^i}+\frac{\pi}{4}\right)\right] \\ \rho^i(z) = \rho_1^i\left[1-\cos\left(\frac{\pi z}{2h^i}+\frac{\pi}{4}\right)\right] + \rho_0^i\cos\left(\frac{\pi z}{2h^i}+\frac{\pi}{4}\right) \\ \quad = \rho_1^i\left[1-e_m^i\cos\left(\frac{\pi z}{2h^i}+\frac{\pi}{4}\right)\right] \end{cases} \tag{2}$$

where E_1^i and E_0^i are the maximum and minimum Young's moduli of the ith FGP beam element, respectively; $G_1^i = E_1^i/(2+2\nu_i)$ and $G_0^i = E_0^i/(2+2\nu_i)$ are the maximum and minimum shear moduli of the ith FGP beam element, respectively; ν_i denotes the Poisson's ratio of the ith FGP beam element; Without loss of generality, the Poison's ratio (i.e., ν_i) of the ith FGP beam element is assumed to be a constant [15–17]. ρ_1^i and ρ_0^i are the maximum and minimum densities of the ith FGP beam element, respectively; $e_0^i = 1-E_0^i/E_1^i$ and $e_m^i = 1-\rho_0^i/\rho_1^i$ are the porosity coefficients of the relative Young's modulus and density of the ith FGP beam element which are also governed by the relationship of an open-cell foam as [20,21]:

$$\frac{E_0^i}{E_1^i} = \left(\frac{\rho_0^i}{\rho_1^i}\right)^2 \Rightarrow e_m^i = 1-\sqrt{1-e_0^i} \tag{3}$$

and $-\frac{h^i}{2} \leqslant z \leqslant \frac{h^i}{2}$, h^i denotes the thickness of the ith FGP beam element. In order to achieve a more effective illustration of the effects of different porosity distributions, the variation of the Young's modulus of a generic FGP beam with a rectangular cross-section under two different material constitutive relationships are depicted in Fig. 1.

Within the SMCR model, the material properties of the FGP beam can have a symmetric profile about the middle plane of the thickness direction. The maximum material properties are being located at the two extreme layers (e.g., top and bottom layers) of the beam and the minimum material properties located at the neutral axis of the FGP beam. On the other hand, the MMCR model is governing the material properties in a monotonic increasing fashion from the bottom layer to the top layer. That is, within the MMCR model, the minimum material properties are located at the bottom layer and the maximum properties are graded at the top layer of the beam. Consequently, such capability

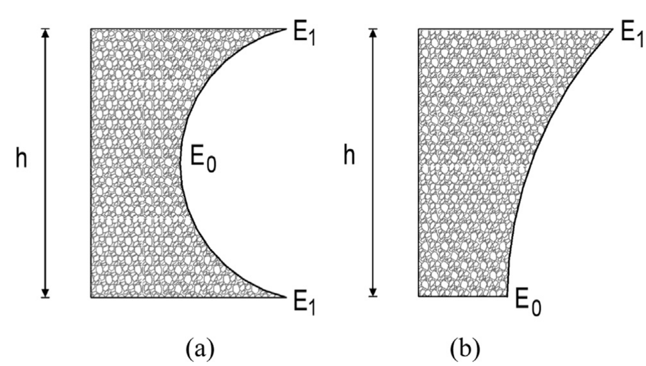

Fig. 1. The Young's modulus of FGP beam with (a) SMCR and (b) MMCR.