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Effectiveness factor of self-compacting concrete in compression for limit analysis of continuous deep beams

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ABSTRACT

The current design codes, such as ACI 318-14, EC2 and CSA23.3-04, in addition to previous research investigations suggested different expressions for concrete effectiveness factor for use in limit state design of concrete structures. All these equations are based on different design parameters and proposed for normal concrete deep beams. This research evaluates the use of different effectiveness factor equations in the upper and lower bond analyses of continuously-supported self-compacting concrete (SCC) deep beams. Moreover, a new effectiveness factor expression is suggested to be used for upper and lower bound solutions with the aim of improving predictions of the load capacity of continuously-supported SCC deep beams. For the range of deep beams considered, the strut-and-tie method with the proposed effectiveness factor formula achieved accurate predictions, with a mean of 1.01, a standard deviation of 6.7% and a coefficient of variation of 6.8%. For the upper-bound analysis, the predictions of the proposed effectiveness factor equation were more accurate than those of the formulas suggested by previous investigations. Overall, although the proposed effectiveness factor achieved very accurate predictions, further validation for the proposed formula is needed since the only data available on continuous SCC deep beams are those collected from the current study.

1. Introduction

Reinforced concrete deep beams are used in construction as load distribution members that receive a relatively high number of small loads, which are transferred to a very limited number of reaction points. They can be found in different civil engineering applications such as stores, hotels, offshore structures, theatres and many others. Although continuously-supported deep beams are commonly used in construction rather than simply-supported ones, all previous investigations have been conducted on simply-supported self-compacting concrete (SCC) beams [1–6]. In contrast, there are no research investigations on continuous reinforced SCC deep beams. This area of research is of special interest due to the high depth of deep beams and congested steel reinforcement, making it difficult for normal concrete (NC) to be properly placed and vibrated. SCC requires no vibration as it can easily flow and be placed under its self-weight with excellent surface finishes and homogenous distribution of concrete within the formwork, to the advantage of durability. However, the lower amount and smaller size of coarse aggregate used in SCC lead to more brittle behaviour and lower shear resistance as cracks can propagate further through the paste or mortar phase before stopped or diverted by a coarse aggregate particle, i.e. less contribution from aggregate interlock [1–6].

The current design codes, namely the ACI Building Code (ACI 318-14)[7], Euro Code 2 (EC2) [8] and Canadian Standard for the Design of Concrete Structures (CSA23.3-04) [9] classify deep beams as a discontinuity region in which the strain distribution is nonlinear. In this case, the classical theory of elasticity is only valid to describe the behaviour of deep beams before cracking. After cracking, however, major redistribution of stresses takes place and the elasticity theory becomes inapplicable [10,11]. Therefore, the current design codes suggest that deep beams should be designed either by nonlinear analysis in which the nonlinear strain and stresses distributions are accounted for or by limit analysis, for example the strut-and-tie model (STM) and mechanism analysis. On the other hand, a number of researchers [12–14] developed a mechanism analysis based on the upper-bound theorem of the plasticity theory to predict the shear strength of deep beams.

The plasticity theory for rigid plastic structures mainly comprises three fundamental theorems, namely the lower-bound, upper-bound and uniqueness theorems. The lower-bound theorem can be developed by considering a safe and statically admissible stress distribution on or within the yield criteria [10,11]. The load obtained by considering equilibrium of internal and external forces of such stress distribution,

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satisfying the static boundary conditions is lower than or at most equal to the collapse load. On the other hand, the upper bound theorem can be derived by considering a kinematically admissible failure mechanism and the load calculated from the energy principle is higher than the collapse load [10,11]. The lower-bound analysis requires finding a load path to transfer forces from the load point to supports [11,15]. However, for complicated loading conditions, it is easier to develop an upper-bound analysis as it just requires a geometrically admissible failure mechanism [13,15]. The uniqueness theorem can be obtained by satisfying the two aforementioned theorems at the same time [10].

Applying the plasticity theory to reinforced concrete structures requires modifying the compressive strength of concrete by a reduction factor, called the effectiveness factor, v. This factor is introduced to overcome the shortcomings of applying the plasticity theory to concrete structures and account for the limited ductility of concrete [13]. It is also considered to account for the compressive strength reduction due to transverse tensile stresses or transverse reinforcement in tension. A number of studies [12,13,16] showed a good correlation between the plasticity analyses of reinforced concrete structures and experimental results when the compressive strength of concrete was reduced by an effectiveness factor.

The main aim of this paper is to evaluate the applicability of both the strut-and-tie model recommended by different design codes and the mechanism analysis proposed by Ashour and Morley [13] to continuously-supported SCC deep beams. The predictions from the two approaches are assessed using different effectiveness factor formulas available in the literature. Moreover, a new formula for the effectiveness factor is proposed for the lower and upper bound solutions in order to achieve more accurate predictions.

2. Experimental program overview

The experimental results of eight continuous SCC deep beams reported by the authors in a previous investigation [17] are used to examine the applicability of the design methods available for NC deep beams to predict the capacity of continuously-supported SCC deep beams. The overall geometrical dimensions along with the reinforcement details for all specimens are presented in Table 1, Figs. 1 and 2. The specimens were tested under a symmetrical two-point loading system, using a loading frame of a capacity of 2500 kN.

The test specimens were made of SCC concrete having a cylinder compressive strength ranged between 31.1 MPa and 50.4 MPa. All the tested beams failed in shear due to a major diagonal crack in the internal shear span started at the mid-depth of the beam and extended along the distance between the edges of the load and intermediate support plates. The significant diagonal crack separated the beam into two concrete blocks: one rotated about the exterior support while the other was fixed over the other two supports. This failure mode was similar to that reported for NC continuously-supported deep beams [13–15]. The tested beams achieved different load capacities depending on their geometrical dimensions, reinforcement arrangement and concrete compressive strength. The results of the cylinder compressive strength, the maximum shear force and the total failure load for each beam are presented in Table 2.

3. Effectiveness factor of concrete

The effectiveness factor of concrete, v, is introduced to overcome the shortcomings of applying the plasticity theorem to reinforced concrete, mainly to account for the limited ductility of concrete [10,16]. The effectiveness factor proposed in the literature mainly depends on concrete properties, geometrical dimensions and reinforcement details [10–16]. There is disagreement among different codes of practice on the value of the effectiveness factor, as shown in Table 3. The ACI 318-14 [7] bases the value of the effectiveness factor on the amount of vertical and horizontal web reinforcement. This means that if the amount of web reinforcement satisfies the requirements presented in Table 3, the value of v is 0.64, otherwise v equals to 0.51. However, the value of the effectiveness factor suggested by the EC2 [8] depends on only concrete compressive strength. On the other hand, the Canadian Standard (CSA23.3-04) [9] recommends a value for the effectiveness factor based on the principal tensile strain of steel reinforcement (ε_1) and the angle between the tie and strut (θ). The value of the principal tensile strain can be approximated as ($\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)/(\tan\theta)^2$), where ε_s is the tensile strain in the ties. For the purpose of design, ε_s can be considered as the yield strain of the steel reinforcement which was obtained by conducting a tensile test on the steel bars used in test specimens. On the other hand, the angle between the strut and tie depends on the a/d ratio ($\tan\theta = d/a$). In the current study, all the beams tested had the same type of reinforcement which means that the value of the tensile strain is constant for all beams while the value of the a/d ratio is different. As a result, the value of the effectiveness factor according to the Canadian Standard can be calculated based on the value of the a/d ratio as shown in Table 3.

On the other hand, a large number of research investigations suggested different formulas for v. As shown in Table 4, three equations for v were selected to be used in the analysis presented in this paper. The selection of these formulas was based on the accuracy of the predictions compared to the experimental results in previous investigations. As can be clearly seen from Table 4, the three selected formulas were based on different material and geometrical properties. Neilsen [16] proposed a formula for v based on the value of f_c. The value of v resulting from this formula ranges from 0.3 to 0.8 for a concrete strength up 100 MPa. However, Vecchio and Collins [18] considered v as a function of concrete strength and principal tensile and compressive strains in steel reinforcement. This formula was modified by Yang and Ashour [15] to reflect the size effect as shown in Table 4. It should be noted that this formula was proposed for the upper-bound analysis and it results in low effectiveness factor values. Another formula was proposed by Warwick and Foster [19], which considers the effect of a/d ratio in addition to the concrete strength with an upper limit of 0.85.

Table 1
Geometrical dimensions and reinforcement details of the beams tested by Khatab et al. [17].

Beam no.	h (mm)	d (mm)	L (mm)	Longitudinal reinforcement ratio (%)		Web reinforcement ratio (%)	
				Bottom	Top	Vertical	Horizontal
B1	600	560	2750	0.67	0.67	–	0.3
B2				0.67	0.67	0.3	–
B3				0.67	0.67	0.3	0.3
B4				0.67	0.67	0.3	0.6
B5				0.67	0.67	0.6	0.3
B6				1.10	1.10	0.3	0.3
B7	300	260		1.42	1.42	0.3	–
B8				2.37	2.37	0.3	–