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Smoothing evolutionary structural optimization for structures with displacement or natural frequency constraints

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ABSTRACT

The *Smoothing Evolutionary Structural Optimization* (SESO) technique was extended to solve 2D elastic problems with constraint of displacements or natural frequencies. At the end of each finite element analysis, a scalar representing the sensitivity due to the removal of an element is calculated. Thus, the elements that have the lowest values are removed from the structure, while the displacements in prescribed locations are kept inside of limits stated or the first frequencies are maximized. The proposed technique proved to be adequate and efficient in the execution of shape and topological optimization.

1. Introduction

Topology Optimization (TO) can significantly improve the performance of structures for many engineering applications. It has been exhaustively studied and various topology optimization methods have been developed over the past few decades. The main optimization techniques include: (a) Homogenisation method - Has been introduced by Bendsøe and Kikuchi [3], which describes the amount of material (i.e. density ρ) at each point of the design domain. Typically, this problem is represented by an initial fixed domain design that is discretized with a finite element mesh. Moreover, the use of multiple design variables for each element increases the computational cost of this method, (b) SIMP method - In order to overcome the difficulties associated with the homogenization method, Bendsøe [4] proposed a density based approach, also called as the SIMP (solid isotropic material with penalisation) method, Rozvany et al. [40]. SIMP method is based on the idea of using an isotropic material within each element of the FE model and is assumed to be a function of the penalized material density, described by an exponent power. SIMP can generate results with checkerboard patterns. Secondly, as it results in mesh dependent optimal solutions, to avoid the occurrence of checkerboard formations and the mesh dependency issues filtering techniques can be used according to Sigmund and Petersson [41] and (c) ESO method - Evolutionary structural optimization is one of most popular techniques for topology optimization, Xie and Stven [52,52,36], Tanskanen [50]. The ESO method was first proposed by Xie and Steven in the early 1990s [52] and has since been continuously developed to solve a large variety of topology optimization problems [56].

Ghabraie [15] presents an approach in which the ESO can be mathematically proved (justifiable), but also discusses that such an approach can result in an inefficient local optimal. A bidirectional version of ESO (BESO) was proposed by Querin et al. [37] and Yang et al. [57], which allows the addition of new elements to the parts of the structures. The term SERA (Sequential Element Rejection and Admission) was later proposed by Rozvany and Querin [39] to distinguish this method from Darwinian-based evolutionary methods. In more recent work on BESO, some researchers have adopted a more rigorous approach to clearly indicate the optimization problem based on sensitivity analysis of objective function. For example, Huang and Xie [19–22,25], Ghabraie [14], Ghabraie et al. [16] and Nguyen et al. [33].

The TO with displacement constraint has as basic strategy the control of local displacement at pre-defined locations, usually at the point of loading and in the direction of loading and it can be found in the literature, Liang et al. [27], Rong and Yi [38] and Suresh et al. [48]. Farahpour [13] and Sonmez [47] have used pre-defined displacement constraints applied in specific structural systems, such as truss structures. Zuo and Xie [60] present a global control method for displacements of continuum structures using a new approach to BESO.

Frequency optimization is applicable in many fields of engineering, aeronautical and automotive industries. The works of Tenek and Hagiwara [51] and Ma et al. [32] developed frequency optimization problems using the homogenization method. Kosaka and Swan [26] have used the SIMP method. However, the SIMP model has been proven to be inadequate for frequency optimization due to local artificial

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modes in lower mass areas. Therefore, the modified SIMP model using a discontinuous function was proposed and successfully applied by Pedersen [35] and Du and Olhoff [12] to solve the problems of frequency optimization. Huang et al. [24] developed a modified SIMP model which was able to effectively avoid artificial modes in regions with lower density values. In this model, it combines rigorous criteria for topological optimization in frequencies of continuum structures with the use of a new bidirectional evolution optimization procedure – BESO whose theory and formulation is described in Huang and Xie [19–21]. The ESO/BESO methods were used for frequency optimization by several authors and can mention Xie and Steven [55], Zhao et al. [59] and Yang et al. [57]. Due to the direct elimination of elements of the design domain, the ESO/BESO methods with "hardkill" procedure effectively prevent localized vibration modes from occurring. The deficiencies pointed out by Rozvany and Querin [39] for the methods quoted with "hardkill" procedure also occur in frequency optimization problems. In this sense, it is desirable that the elements of the design variables are not directly eliminated from the domain, unless an equivalence of intermediate elements, i.e., soft elements are equivalent to void elements. In this sense, in the present paper, it is proposed a simple method that solves displacement and frequency optimization problems, using a filter procedure that reduces the presence of the checkboard, monitoring the optimization process that is performed by performance index, which produces lighter structures than other optimization methods. This paper potentializes the SESO technique to solve problems of topological optimization of elastic structures with dynamic loads and multiobjective optimization with displacement and frequency constraints. In this context, topology optimization problem for this purpose is formulated under the finite element (FE) scheme. The sensitivity analysis is carried out on the macro structure and the FE analysis in the macro structure considers the boundary conditions and the external loads. To evaluate the effect of removing the elements of the structure, a sensitivity number has been defined, as presented by Zhao et al. [59], Chu et al. [9–11], Yang et al. [57], Liang et al. [29] and Liang [30]. Then, a series of elements with the lowest sensitivity numbers will be deleted from the structure. The optimum structure configuration will be obtained by repeating the FE analysis cycle, the calculation of a sensitivity number and removal of elements until the specified displacements reach their prescribed limits or the natural frequencies are maximized. Thus, a variant of ESO called SESO is applied, created by Simonetti [42] and expanded by Simonetti et al. [43–45], Simonetti et al. (2015), Simonetti et al. [46] and Almeida et al. [2,1], which are based on a smoothing of the heuristic of removal of the ESO method. Theoretically, one cannot guarantee that such the evolutionary procedure can always produce the best solution. However, the SESO technique provides a useful tool for engineers and architects who are interesting in exploring structurally efficient forms and shapes during the conceptual design stage of a project.

The remainder of the paper is organized as follows: Section 2 deals with the formulation of the problem SESO. In Section 3 formulates the basic optimization problem for frequency and displacement. In Section 4, sensitivity numbers in frequency and displacement. Section 5 presents the numerical results from the proposed SESO method in displacement and frequency. Concluding remarks are made in Section 6.

2. Formulation of the SESO problem

The minimization problem can be expressed in the following standard form:

$$\begin{aligned} &\text{minimize} f(\{x\}) \\ &\text{subject to} g_j(\{x\}) \leqslant 0 \;\; with \;\; j = 1,2,...,k \\ &\qquad h_j(\{x\}) = 0 \;\; with \;\; j = 1,2,...,l \\ &\qquad x_j^i \leqslant x_j \leqslant x_j^f \;\; with \;\; j = 1,2,...,m \end{aligned} \tag{1}$$

where $f(\{x\})$ is the objective function and $\{x\}$ is the vector of the design variables. $g_j(\{x\})$ and $h_j(\{x\})$ refer, respectively, the inequality constraints and equality with k and l representing the number of inequality and equality, respectively. x_j^i and x_j^f are the lowest and highest value of the design variable x_j and m is the total number of design variables. In this sense, SESO technique has been successful in applying flat elastic problems with stress and strain energy constraints. The optimization via SESO is performed removing p% and returning (1-p%) of the elements for structure, i.e., the element itself, instead associated physical or material parameters, is treated as the design variable. Thus, unnecessary regions have their elements removed so that their corresponding stiffness matrices are eliminated. For this, the elastic modulus of the elements can be chosen as design variables, E_j ($j = 1,2,\ldots,m$), where in its heuristic of removal the SESO presents typical characteristics of the continuous optimization, once the remaining domain contains: removed elements, with null stiffness (minimum), elements that return for the structure, with intermediate stiffness and elements that remain in the structure, with maximum stiffness. Thus, it is possible to write the design variables as:

$$0 \leqslant \alpha * E_j \leqslant E_j^{project} \;\; with \;\; j = 1,2,...,m \tag{2}$$

where m represents the number of elements in the design domain and $E_j^{project}$ indicates the design value of the modulus of elasticity of each element and α a weighting factor that smoothes the elasticity matrix of each element, see Fig. 1.

Therefore, to minimize energy through removing elements, it is clear that the most effective way is to eliminate the elements having the lowest values so that energy growth is minimal. In this sense, optimization problem in energy can be described as follows:

$$\begin{aligned} &\text{Minimize} U(X) = \frac{1}{2}u^T K u = \sum_{1}^{NE} \frac{1}{2}\int_{V_e} \varepsilon_e^T E_e(x) \varepsilon_e dV_e \\ &\text{subject to} Ku = F \\ &\qquad V(X) = \sum_{i=1}^{NE} x_i V_i - \overline{V} \leqslant 0 \\ &\qquad X = \{x_1 \;\; x_2 \;\; x_3 \;\; \ldots \;\; x_N\}, x_i = 1 \; or \; x_i = 0 \end{aligned} \tag{3}$$

where E_e is the elastic matrix of the element, ε_e is the vector of the strains of the element, V_e is the volume of element, NE is the number of finite elements of the mesh, K is stiffness matrix, $Ku = F$ is the equilibrium equation where F is the vector of the loads applied in the structure, x_i is the design variable of the ith element and X is the vector of the design variables.

In this work, the SESO technique is expanded to the optimization problem with frequency and displacement criteria, see section 3. The method has presented results comparable to other different types of methods such as ESO, BESO, SIMP and the Homogenization Method which demonstrates the good performance of the method, through a simple algorithm that can incorporate the self weight of the structure during the optimization process, which is not observed in other methods for the criterion of displacement.

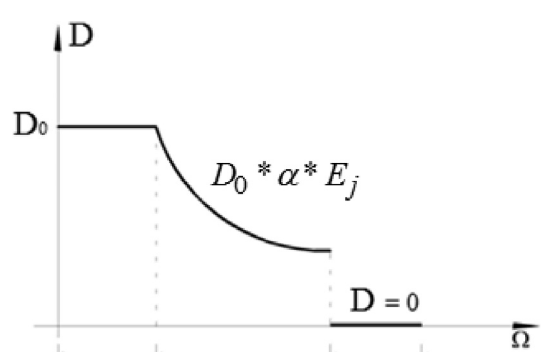

Fig. 1. Smoothing of the volume of the elements removed in iteration i.