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Evaluation of as-installed properties of transformer bushings

Nicholas D. Oliveto*, Andrei M. Reinhorn

Department of Civil, Structural and Environmental Engineering, University at Buffalo, 212 Ketter Hall, Buffalo, NY 14260, USA

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ABSTRACT

High voltage transformers are essential parts of the electrical distribution grid. Severe failure of entire grids can occur during earthquakes, when transformer bushings fail due to structural dynamic mismatch to the seismic demands. Proper consideration of the fundamental frequency of vibration, which depends on the flexibility of the cover plate of the transformer to which they are connected, is therefore crucial for determining the seismic response of bushings. A simplified method is developed in this work for the evaluation of the "as-installed" fundamental frequency of transformer bushings. Such bushings are modeled as cantilever beams with distributed mass and elasticity, and an additional rotational spring is introduced at the base, to account for the flexibility of the cover plate of the transformer. A simple yet efficient expression is derived for the as-installed frequency, based on the Southwell-Dunkerley method. The solutions require the knowledge of the bending rigidity of the bushing and the (out-of-plane) rotational stiffness of the cover plate. The evaluation of both these quantities is presented. While analytical solutions for the (out-of-plane) rotational stiffness of circular plates are well known, solutions for rectangular plates have not yet been addressed. A semi-empirical-numerical solution is suggested, based on finite element models and analytical expressions derived by force-fitting the "circular plate solution" to the numerical analyses. The results yielded by the proposed method are compared with experiments on real bushings.

1. Introduction

Substation transformers, and bushings, are essential components of power delivery systems. Although they are designed and manufactured separately, their dynamic behavior cannot be analyzed independently. During recent earthquakes, both in the United States and abroad, bushings have sustained severe damage and failures, compromising the functionality of the entire transformer-bushing system. This non-satisfactory seismic performance is an indication of potential flaws and limitations in the current design concepts, and testing-qualification procedures of transformer bushings, embodied in IEEE Standard 693-2005 [1]. It is current practice to evaluate the seismic demands on bushings based on the fixed base frequency, therefore ignoring the (out-of-plane) flexibility of the cover plate of the transformer to which the bushings are connected. However, depending on the properties of the bushings, and on the out-of-plane stiffness of the cover plate, the "as-installed" frequency that dominates bushing responses can be considerably lower than the fixed base one, leading to seismic demands that can be much higher than those considered for design. Experimental studies [2–14] show that bushings with same construction appear to have varying dynamic properties when installed on the roof of transformers, dependent on the transformer cover construction. Some of the experimental results are presented in Section 6 of this paper, and are included in Table 3 for sake of comparisons.

The objective of the present paper is to propose a simplified method for the evaluation of the "as-installed" frequency of transformer bushings. For this purpose, the bushings are modeled as simple cantilever beams with uniformly distributed mass (m) and elasticity (EI), as shown in Fig. 1. The out-of-plane flexibility of the cover plate is accounted for by introducing a rotational spring of stiffness k at the base. The frequencies and modes of vibration of such systems, as also buckling loads and modes, can be determined by analytical methods that are well-established in the literature [15–17]. However, these require the solution of an eigenvalue-eigenfunction problem, which may not be appealing for design purposes. An approximate solution for the fundamental as-installed frequency is derived herein, based on the Southwell-Dunkerley method [18].

The exact and approximate solutions both require knowledge of the bending rigidity of the bushing and out-of-plane rotational stiffness of the cover plate. The former is generally difficult to evaluate based on the variable geometry and material of the bushings [5,7]. The cover plate is usually supported by a rectangular grid of beam-like stiffeners, and obtaining its out-of-plane stiffness requires the solution of a boundary value problem involving the interaction between the bushing

* Corresponding author.
E-mail address: noliveto@buffalo.edu (N.D. Oliveto).

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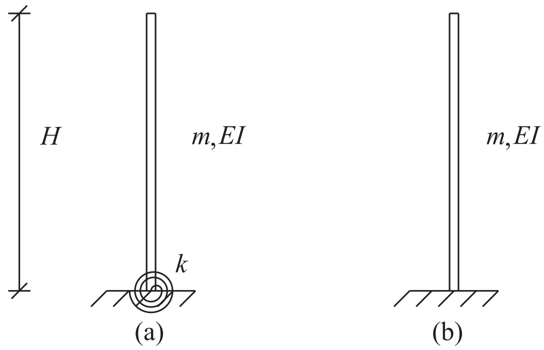

Fig. 1. Models used to describe the dynamic behavior of transformer bushings: (a) uniform flexible base cantilever, (b) uniform fixed base cantilever.

and the cover plate. The bending rigidity of the bushings is herein evaluated by inverting the expression for the fundamental frequency of the fixed base cantilever, which is determined by experiments [2,3,6,7,12,19]. Analytical solutions for the out-of-plane rotational stiffness of the cover plate are available only for circular plates [16,20]. For square and rectangular plates, as in the case of a supported grid, the same formulations require to express the solution in the form of an infinite series. However, application of the boundary conditions, needed to determine the coefficients of the series, leads to an ill-conditioned algebraic problem. For this reason, finite element models (FEM) in ABAQUS [21] are developed in this work, to solve the boundary value problem and determine the rotational stiffness of the cover plate. The accuracy of the finite element results is first tested against the analytical solution for circular plates. The same convergence criteria are then applied to the square and rectangular plates. Analytical expressions, obtained by fitting the results of the finite element analyses, are then derived for the evaluation of the rotational stiffness of the cover plate. Finally, numerical applications are presented, where the simplified procedure is used and the results compared to those obtained by experiments carried out at the Structural Engineering and Earthquake Simulation Laboratory (SEESL) at the University at Buffalo, USA.

2. Exact frequencies of vibration of beams with distributed mass and elasticity

The frequencies and modes of vibration of beams with distributed mass and elasticity on a rigid base can be determined by well-known analytical methods, and are readily available in the literature [15,16]. The natural frequencies of flexible beams on a flexible base can also be obtained by solving for the roots of the so-called frequency equation, which depends on the boundary conditions of the beam. In the following subsections, expressions for the natural frequencies of vibration are presented, for boundary conditions that concern the evaluation of the dynamic response of transformer bushings.

2.1. Fixed base cantilever

Transformer bushings can be modeled as cantilever beams mounted on a flexible base represented by the cover plate, Fig. 1(a). As a first step, the fundamental frequency of the fixed base cantilever, Fig. 1(b), is re-derived here for completeness [15,16]. The n frequencies of vibration are derived by solving the following frequency equation:

$$1 + \cos(\beta H)\cosh(\beta H) = 0 \tag{1}$$

where

$$\beta^4 = \frac{\omega^2 m}{EI} \text{ and } \omega = 2\pi f \tag{2}$$

Eq. (1) can be solved numerically, and its roots $\beta_n H$ used in Eq. (2) to obtain the natural frequencies of the system.

Herein we are mainly interested in the fundamental frequency, which is obtained by substituting the smallest root of Eq. (1) ($\beta_1 H = 1.8751$) into Eq. (2) to get [15,16]:

$$f_{1,f} = \frac{1}{2\pi}\frac{3.516}{H^2}\sqrt{\frac{EI}{m}} \tag{3}$$

2.2. Flexible base cantilever

The flexible base mounted bushing is obtained by adding the flexibility of the cover plate which is accounted for by considering a rotational spring of stiffness k, as shown in Fig. 1(a). The frequency equation is in this case:

$$1 + cC - \beta H\sigma(sC - cS) = 0 \tag{4}$$

where

$$\sigma = EI/kH \tag{5}$$

$$s = \sin(\beta H);\ c = \cos(\beta H);\ S = \sinh(\beta H);\ C = \cosh(\beta H) \tag{6}$$

Eq. (4) can be solved numerically, and its roots $\beta_n H$ used in Eq. (2) to obtain the natural frequencies of the system. Again, the fundamental frequency is obtained by substituting the smallest root of Eq. (4) into Eq. (2) to get:

$$f_{1,k} = \frac{1}{2\pi}\frac{(\beta_1 H)^2}{H^2}\sqrt{\frac{EI}{m}} \tag{7}$$

The roots of Eq. (4), $\beta_n H$, depend on the stiffness of the cover plate, k, through the dimensionless parameter $\sigma = EI/kH$. The influence of the rotational spring at the base on the fundamental frequency is shown, in Fig. 2, by plotting the ratio of the frequency of the cantilever mounted on a semi-rigid base (represented by the spring of stiffness k) to that of the fixed base cantilever, as a function of σ. A reduction in "as-installed" frequency due to the flexible base is seen to be particularly important when σ becomes large, obviously for a base characterized by small k, or for stiff bushings characterized by large EI, small H, or both.

3. Approximation to the fundamental frequency using the Southwell-Dunkerley method

A very simple approximation to the exact frequency ratio curve plotted in Fig. 2 may be obtained using the Southwell-Dunkerley method [18], as presented below. The method provides a lower limit approximation while maintaining simplicity. The cantilever bushing on flexible base is treated as the sum of a flexible beam (*EI*) on fixed base and a rigid beam (EI = ∞) on flexible base (*k*), both beams having the

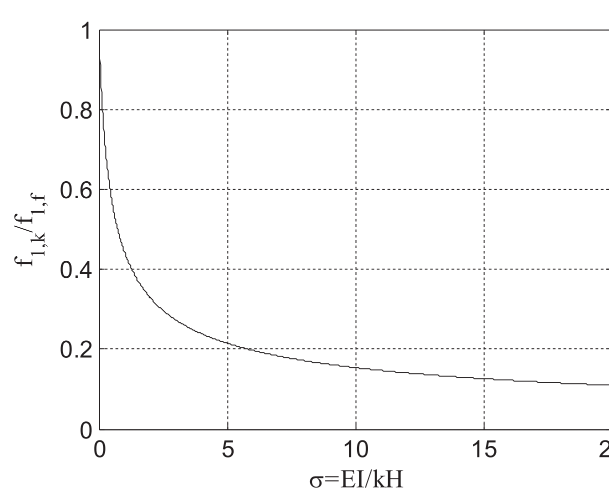

Fig. 2. Influence of the base rotational spring on the "as-installed" frequency of bushing.