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Behaviour under long-term loading of externally prestressed concrete beams

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ABSTRACT

External posttensioning is a useful technique both for strengthening damaged beams, and for the construction of new girder bridges. In an externally posttensioned member, the prestressing tendons are located outside the cross section and connected to the structure by anchorages and deviators, that are the only points where compatibility occurs (at least in the direction orthogonal to the beam axis). This fact implies that a simply supported, externally prestressed beam is a redundant structure.

Two numerical methods able to describe the time evolution of both the stress distribution and the displacements of a simply supported concrete beam externally prestressed are introduced and discussed. The distinction between them lies in the way they solve the Volterra integral that is involved in the solving system: one of them performs a numerical solution (almost exact) of this integral whereas the second one adopts an approximate solution.

The example presented makes it possible to verify the precision of the approximate method when compared to the almost exact one.

1. Introduction

External prestressing is a post-tensioning method in which tendons are placed on the outside of the structural element. Its main advantages with respect internal post-tensioning (with the tendons bonded to concrete) are (see [1,2]):

- removal and replacement of one of the tendons at a time is possible because of the absence of bond between the tendon and the concrete substructure so that the structural element could be retrofitted in the event of deterioration. When dealing with bridges in urban areas this advantage is crucial because so doing traffic disruption is avoided.
- adjustment and control of the tendon forces is simple and therefore errors in the computation of the prestressing losses related to creep of concrete can be corrected.
- inspection of the tendons is feasible, and access to anchorages is usually easy.
- the designer has more freedom in selecting the shape of the concrete cross-section.
- concrete cross-section could be made thinner (especially the webs) due to the partial or full elimination of internal tendons (dead weight reduction).
- friction losses are significantly reduced because external tendons are linked to the concrete substructure only in the deviators and in the anchorages.

In addition, a lot of engineering structures that at service suffer from fatigue or damage due to various causes such as earthquakes, overloads and structure aging can be effectively strengthened by means of external prestressing (see for instance [3–8]).

On the other end the main disadvantages of external prestressing are:

- the contribution of external tendons to flexural strength is reduced compared to internal grouted tendons. This is because the stress variation in the tendons between the cracking load and ultimate does not depend only on the local curvature in the critical section (as is done for internal bonded tendons). For this load range, the behaviour of external tendons is less effective than that of internal bonded tendons (see for instance [9]).
- when dealing with box girders, the actual eccentricities of external tendons may be smaller compared to internal tendons because they are placed in the inner cavity. This is usually not true when dealing with I beams externally prestressed, because in this case the tendons can be placed even under the lower face of the beam.
- the effective depth of external tendons varies with the development of the member deflection, causing what is called second order effects (see [10–12]).
- a consequence of what is said in the previous point is that at ultimate load frequently the tendons are still in the elastic range (see for instance [13,14,3]). Therefore failure usually occurs with little warning.

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Notation

A_c	area of the cross-section of the beam
A_{pi}	area of the ith tendon
$E_c(t)$	elastic modulus of concrete at time t
E_p	elastic modulus of the ith tendon
$\underline{F}_p(t)$	vector of the tensile stress resultant in the n tendons
$J(t, t_0)$	creep function of concrete
$J_{yy,c}$	second moment of area of the cross-section of the beam
$M_c(z)$	bending moment in cross-section z of the homogeneous concrete beam
$N_c(z)$	axial force in cross-section z of the homogeneous concrete beam
n	number of the tendons
$\frac{1}{r}(z,t)$	curvature of cross-section z of the homogeneous concrete beam, at time t
$R(t,t_0)$	relaxation function
$\underline{S}_c(z,t)$	vector of the internal actions acting at generic time t, in cross-section z of the homogeneous concrete beam
$\underline{S}_{c0}(z)$	vector of the internal actions due to the weight of the beam (applied at time t_0) acting at generic time t, in cross-section z of the homogeneous concrete beam
$\underline{\underline{S}}_{c1}(z)$	vector of the internal actions due to each single tendon stressed with a unit force
t	actual time
t_0	time of first loading
$V_c(z)$	shear in cross-section z of the homogeneous concrete beam
y_G	position of the centroid of the concrete cross-section
z	longitudinal axis of the beam
$\alpha_{i,\ell}$	slope of the ℓth segment of the piecewise polygonal shape of the ith tendon
$\chi(t,t_0)$	aging coefficient
$\underline{\Delta}$	"instantaneous change"
$\varepsilon_0(z,t)$	strain along the z axis, in cross-section z of the homogeneous concrete beam, at time t
$\varepsilon_{sh}(z,t)$	shrinkage of the homogeneous concrete beam, at time t, in cross-section z
$\varphi(t,t_0)$	creep coefficient of concrete

Although creep losses can be recovered, or errors in their computation can be corrected, it would anyway be helpful for the designer to get a reliable forecasting of the time evolution of the stress distribution in concrete beams externally prestressed (see for instance [15]). Nevertheless, even when dealing with simply supported structural elements this task is particularly complex because of the internal redundancy of the structure. In fact in this case the analysis of externally prestressed concrete beams is member dependent, rather than section dependent as in the case of ordinary prestressed concrete beams (bonded prestressing), due to strain incompatibility between external tendons and adjacent concrete. In short strain in every tendon is almost constant over its length and depends on the overall behaviour of the beam.

This paper suggests two numerical methods able to solve the problem. The two methods differ from one another only in the way they solve the time integrals. The first of these methods is the more sophisticated and cumbersome, and will be used especially to verify the precision of the second one which implies a simpler, but approximate solution.

A numerical example will highlight the error level of the simplified method and generally speaking the potential of the proposed approaches.

2. The heterogeneous structural member

Let us consider the case of a simply supported homogeneous, concrete beam stressed by its weight (that rests on the mould till the prestressing action detaches the beam from the scaffolding and self-weight starts mobilizing, or till the scaffolding is disassembled, if prestressing was not adequate to move the beam up and mobilize the self-weight) and by external prestressing. This assumption implies that the contribution of the rebars to the overall behaviour of the beam will be neglected, but this simplification should not be very significant because the area of the bonded longitudinal steel reinforcement is normally relatively small and values from 0.004 to 0.010 times the area of the cross-section are acceptable (weak heterogeneity of the concrete cross-section). The structural member is then the coupling of two substructures, i.e. the concrete beam and the external cables, that interact only in the anchorages and in the deviators (see Fig. 1).

The tensile stress in the tendons obviously depends on friction in the deviators, that however is negligible because usually these tendons are made of strands (installed either singly or in bundles to create the tendon) coated with grease (or wax) and enclosed in a plastic sheath. Therefore the tensile stress in the external tendon is almost constant all over its length.

The internal actions acting at generic time t, in cross-section z of the homogeneous concrete beam (index c):

$$\underline{S}_c(z,t) = \begin{vmatrix} N_c(z,t) \\ M_c(z,t) \end{vmatrix} \tag{1}$$

are the sum of those due to the weight of the beam (placed into vector $\underline{S}_{c0}(z)$) plus those due to prestressing that in their turn are the product of the internal actions due to each single tendon stressed with a unit force (placed in matrix $\underline{\underline{S}}_{c1}(z)$) and the actual value of the tensile stress resultant in the n tendons (contained in vector $\underline{F}_p(t)$ that is the unknown of the problem), that is:

$$\underline{S}_c(z,t) = \underline{S}_{c0}(z) + \underline{\underline{S}}_{c1}(z)\cdot\underline{F}_p(t) \tag{2}$$

Matrix $\underline{\underline{S}}_{c1}(z)$ changes along the axis of the beam at the corners of the piecewise polygonal shape of the tendons (see Fig. 2). If it is assumed (for sake of simplicity) that the deviators and the anchorages are placed for all the tendons at the same coordinates along the longitudinal axis of

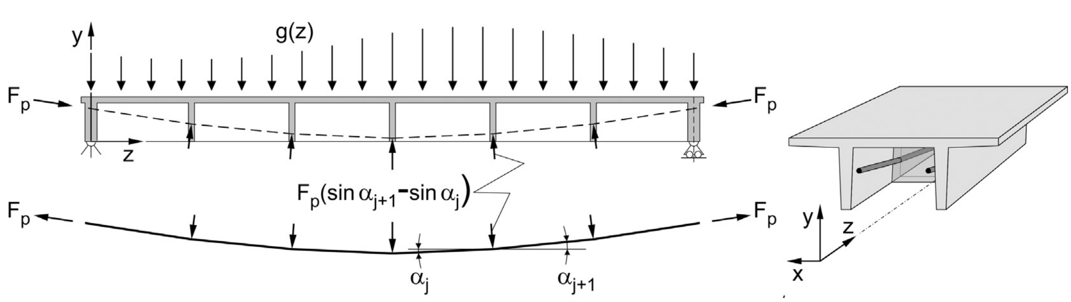

Fig. 1. Uncoupling of the simply supported, concrete beam.