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On collapse of non-uniform shallow arch under uniform radial pressure

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ABSTRACT

Analytical investigation of collapse of a pinned non-uniform shallow circular arch under uniform radial pressure is presented in this paper. The non-uniformity is introduced by dividing the arch into three regions of constant stiffness. The equilibrium equations are obtained by using least potential energy principle. By proper non-dimensionalization, the presented solution is independent of total length of the arch. And two modified slenderness parameters are identified which enables the result to be valid for any symmetric cross-section shape. Parametric study on different geometric parameters has been carried out on the snap-through buckling load and anti-symmetric bifurcation load. The validity of the analytical result is verified by comparison with FEA results. Criteria concerning four deformation modes are identified. We show there is an equal potential energy load for some non-uniform shallow arches by a straightforward deduction, the existence of which load is a necessary condition for the buckle propagation of a corresponding long shallow panel. Finally two limiting cases (rigid center case and rigid end case) with extreme non-uniformity are analytically studied by using augmented potential energy with Lagrangian multipliers. For rigid center case, closed-form condition for the possible occurrence of symmetric snap-through is presented. For rigid end case, an asymptotical analysis leads to a somewhat surprising critical value of slenderness which makes the snap-through buckling just possible. This paper intends to improve the understanding of the effect of non-uniformity on collapse of shallow arch under radial uniform pressure.

1. Introduction

In engineering, shallow arch has many applications such as bridges. The pre-buckling behavior of shallow arch is highly non-linear in contrast to that of the deep arches such as rings. Thus buckling analysis by assuming linear pre-buckling behavior which is often adopted for deep arch analysis is not accurate for shallow arch [1]. The first analytical exact result concerning a shallow arch with two ends fixed under uniform radial pressure or a central concentrated force was obtained by Schreyer and Masur [2]. But in this early research rectangular cross-section was assumed. Early researchers also include Timoshenko and Gere [3], Gjelsvik and Bodner [4], Plaut [5] and so on. Gjelsvik and Bodner [4] conducted an experiment for shallow arch under a concentrated load, derived approximate solutions and considered the effect of small imperfections. More recently, different boundary conditions' effect was intensively studied. To allow description for arbitrary cross-section shape, Bradford et al. [1] introduced a modified slenderness parameter. For example, Pi and Bradford[6] studied the non-linear buckling and postbuckling of arches with unequal rotational end restraints under a central concentrated load. They showed that bifurcation was not possible with unequal rotational end restraints. Pi et al. [7,8] studied analytically the in-plane buckling and post-buckling of shallow arch with equal rotational end restraint. It should be pointed out that more recently Bateni and Eslami [9,10] studied the stability of shallow arch made of functional graded material FGM under radial uniform pressure or a concentrated load analytically by adopting the same shallow arch strain measure as in literature[2]. A new analytical method for heterogeneous curved beams under a concentrated load has been derived by Kiss ad Szeidle [11]. Schreyer and Masur [2] proved that there was an equal potential load. And by this formulation, Power and Kyriakides [12,13] conducted a buckling propagation analysis of a long shallow circular panel under external pressure and showed that the equal potential energy load obtained by Schreyer and Masur [2] agreed well with the quasi-static buckle propagation pressure by a so-called Maxwell construction. They also conducted experiments to verify this.

From the literature, uniform shallow arch has been comprehensively studied, while only a few researches have been conducted for non-uniform shallow arch. Very recently Tsiatas and Babouskos [14] used an interesting numerical method named "analog equation

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Nomenclature

R	radius of shallow arch
X,Y	spatial Cartesian coordinate
A_1,A_2	area of cross-sections for center region and end regions respectively
I_1, I_2	moment of inertia of cross-sections for center region and end regions respectively
E	elastic modulus
s	angular coordinate for shallow arch defined in Fig. 1
$\theta_1,\theta_2,\theta_3$	angles defined in Fig. 1 and $\theta_3 = \theta_1 + \theta_2$
λ	$\lambda = \theta_1/\theta_3$
α_1,α_2	non-uniformity parameters defined as $\alpha_1 = EA_2/EA_1$ and $\alpha_2 = EA_1/EA_2$
β_1,β_2	non-uniformity parameters defined as $\beta_1 = EI_2/EI_1$ and $\beta_2 = EI_1/EI_2$
γ_1,γ_2	slenderness parameters defined as $\gamma_1 = \sqrt{EI_1/EA_1R^2}$ and $\gamma_2 = \sqrt{EI_2/EA_2R^2}$
λ_{s1}, λ_{s2}	modified slenderness parameters defined as $\lambda_{s1} = \theta_3^2/\gamma_1$, $\lambda_{s2} = \theta_3^2/\gamma_2$
q	magnitude of uniform pressure in unit N/m
N_1,N_2	membrane force for center region and end regions respective
N	membrane force defined as $N = N_1 = N_2$ (since $N_1 = N_2$)
μ_1,μ_2	$\mu_1^2 = NR^2/EI_1$, $\mu_2^2 = NR^2/EI_2$
p	$p = (qR-N)/N$
$\overline{p}$, $\widetilde{p}$	$\overline{p} = qR^3\theta_3^2/EI_1$, $\widetilde{p} = qR^3\theta_3^2/EI_2$
$\hat{w}_1,\hat{w}_2$	circumferential displacements for center region and end regions respectively (positive if in the direction of increasing s)
$\hat{v}_1$, $\hat{v}_2$	radial displacements for center region and end regions respectively (positive if pointing to origin)
w_1,w_2, v_1,v_2	$w_1 = \hat{w}_1/R$, $w_2 = \hat{w}_2/R$, $v_1 = \hat{v}_1/R$ and $v_2 = \hat{v}_2/R$
o	centroid point of cross-section
$\bar{x},\bar{y}$	coordinates of cross-section defined in Fig. 1
ε_{1m}, ε_{2m}	membrane strain for center region and end regions respectively (see Eq. (2.1))
$\varepsilon_{1b},\varepsilon_{2b}$	bending circumferential strain for center region and end regions respectively (see Eq. (2.2))
$\varepsilon_1,\varepsilon_2$	total circumferential strain for center region and end regions respectively defined as $\varepsilon_1 = \varepsilon_{1m} + \varepsilon_{1b}$, $\varepsilon_2 = \varepsilon_{2m} + \varepsilon_{2b}$
$()'$, $\delta()$	derivative with respect to s and first variation of () respectively
Π, Π^*	system's potential energy and augmented potential energy (see Eq. (3) and Eq. (39))
λ_1, λ_2	Lagrangian multipliers
x^*, x^{**}, $\tilde{x}$	least positive root of some equations

method" which is one of integral techniques to solve differential equations to study the buckling of non-uniform shallow arch. Recently Yan et al.[15,16], Ye et al. [17], Shen et al. [18] and Xue and Fatt [19] conducted comprehensive analysis of a cylinder with piecewise constant stiffness. Very recently, the buckling of non-uniform shallow arch under a central concentrated load has been studied by Yan et al. [23]. There is a lack in exact analysis of shallow arch with non-uniformity under radial pressure. This paper presents a comprehensive theoretical analysis on non-uniform shallow arch under uniform radial pressure and non-uniformity is characterized by dividing the arch into three regions each of which has constant stiffness. This research should enhance the understanding of the effect of non-uniformity of shallow arch concerning the stability problems under radial loads.

The structure of this paper is as follows. Model description is in Section 2. Theoretical formulation by least potential energy is presented in Section 3, and detailed analysis and discussion are found in Section 4. In Section 5, we present a straightforward deduction to show the existence of an equal potential energy load of non-uniform shallow arch. Finally in Section 6, two limiting cases of extreme non-uniformity are studied in detail with the introduced augmented potential energy, and some criteria concerning the four deformation modes are identified.

2. Model description

The shallow arch has non-uniformity represented by three piecewise regions, and see Fig. 1. The shallow arch geometry is symmetric about axis Y. For the central region, the cross-section area ($-\theta_1 \leqslant s \leqslant \theta_1$) is denoted A_1. The end regions including the left region ($-\theta_3 \leqslant s \leqslant -\theta_1$) and the right end region ($\theta_1 \leqslant s \leqslant \theta_3$) are symmetric with each other and have the same cross-section area A_2. At two ends, pinned-pinned boundary condition is assumed. A uniform radial load q in unit N/m is imposed. The radial displacements (positive if pointing to the origin point O) are denoted $\hat{v}_1$ and $\hat{v}_2$ for center region and end regions, and $\hat{w}_1$ and $\hat{w}_2$ (positive in the direction of increasing s) are tangential displacements for the center region and two end regions respectively. The cross-section shape is arbitrary with only one restriction that it is reflectively symmetric as in Fig. 1(b), in which figure o is the centroid point of the cross-section. It is the paper's objective to investigate the effect of non-uniformity on the buckling behavior of shallow arch.

3. Theoretical formulation

3.1. Governing differential equations for symmetric deformation

By the least potential principle, the differential equations are derived via variation of calculus. The shallow arch strain measure [20] is used as:

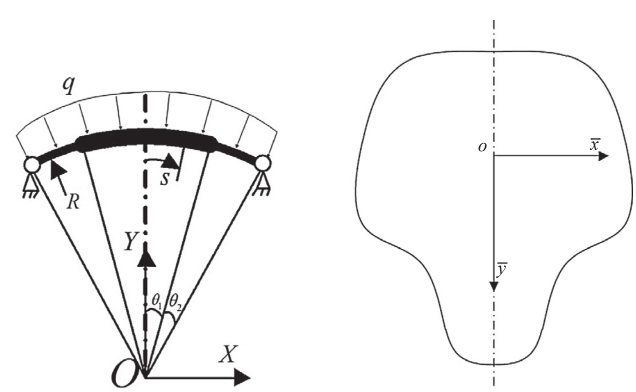
$$\varepsilon_i = \varepsilon_{im} + \varepsilon_{ib} \tag{1}$$

where ε_i, ε_{im}, ε_{ib} denote the total circumferential strains, circumferential membrane strains and bending strains respectively with $i = 1$ denoting values for center region and $i = 2$ denoting values for end regions.

We should emphasize here that the shallow circular arch's deformation is restricted to be in-plane. Thus no torsional and lateral deformations are allowed. And Euler-Bernoulli assumption is used and thus no shear strain is considered. The shallow arch strain expressions are presented as [1]:

$$\varepsilon_{im} = w_i'-v_i + 1/2(v_i')^2 \tag{2.1}$$

$$\varepsilon_{ib} = -\bar{y}v_i''/R \tag{2.2}$$



(a) shallow arch (b) symmetric cross-section

Fig. 1. Non-uniform shallow arch under uniform radial load q.