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Numerical aspects of determination of natural frequencies of a power transmission line cable equipped with in-line fittings

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ABSTRACT

In the analysis of Aeolian vibration response of electric power transmission lines, the modified energy balance method (MEBM) is often used. The first and crucial step in applying the MEBM is an accurate determination of the natural frequencies and modes of the system, or in other words, the eigenvalues and eigenvectors of the system matrix. In this paper an efficient numerical procedure that searches for frequency parameters s that make the system matrix $J(s)$ of transmission line cable singular is considered. A realistic case where the cable is equipped with in-line fittings such as Stockbridge vibration dampers and aircraft warning spheres is taken into account. It is shown that the rank of the considered system matrix $J(s)$ is either full, or the full rank minus one. From a numerical point of view, this is an important property of the system matrix since multiple eigenvalues without a full set of eigenvectors, i.e., with Jordan blocks of order greater than 1, are very sensitive to small perturbations. The developed numerical procedure, which is easily parallelizable, consists of the hybrid minimization method paired with the Singular Value Decomposition (SVD) for the detection of the singularity of the matrix. Presented numerical examples illustrate advantages of the numerical procedure proposed.

1. Introduction

Intense vibrations can be observed in electric overhead transmission lines, submarine periscopes, tall chimneys and other similar objects, when certain conditions are fulfilled regarding the interaction of the fluid flow and the dynamics of the structure. For example, if one considers a fluid flowing past a cylinder, a regular pattern of alternating vortices, the so-called von Kármán vortices, can occur. These vortices change between clockwise and counter-clockwise rotation direction and produce harmonically varying lift forces on the cylinder perpendicular to the fluid velocity. If the Reynolds number (Re) spans from 60 to 5000, experimental data confirms that a strong regular vortex shedding appears. Vortex shedding means that vortices are created at the back of the body and detach periodically from either side of the body. If the Reynolds number is equal or greater than 1000, we can write the dimensionless frequency of the vortex shedding, through a Strouhal number (St), approximately as

$$\mathrm{St} = \frac{fD}{v}. \tag{1.1}$$

In this equation, f is the frequency of the vortex shedding, D is the diameter of the cylinder, and v is the velocity perpendicular to the longitudinal axis of the cylinder. Being a fully coupled aero-elastic phenomenon, the mechanism of forming and shedding of von Kármán vortices is rather involved [11,12,19,29]. If the frequency of the vortex shedding coincides with a natural frequency of the structure, intense vibrations of the structure can occur.

Vibrations of overhead transmission lines caused by the shedding of von Kármán vortices usually occur in the frequency range of 10–50 Hz for low to moderate winds (1–10 m/s). The first natural frequency of a typical overhead transmission line conductor is of the order of 0.1 Hz. Therefore, the frequency range of 10–50 Hz approximately corresponds to the interval from the 100th to the 500th eigenfrequency of the cable. This means that almost certainly the vortex shedding frequency matches a natural frequency of the cable. This regularly causes vibrations of relatively low amplitudes (up to a conductor diameter). However, the cable undergoes a very large number of cycles. This kind of wind induced vibrations with relatively small amplitudes but with prolonged duration is often referred to as Aeolian vibrations [14,22,25]. Aeolian vibrations cause damage and even structural failure of the cable due to material fatigue and significantly shorten its lifetime [8,1].

Extreme fatigue of cable strands occurs at points where motion of the cable is constrained against transverse vibrations. These points are typically suspension clamps, Stockbridge damper clamps, spacer-

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damper clamps, and aircraft warning sphere clamps. Spans with aircraft warning spheres are prone to intense Aeolian vibrations, especially inside sub-spans in between two adjacent warning spheres. It is often the case that the strains at warning sphere clamps are larger than the corresponding bending strains at the suspension clamps [2,7,22] and are beyond the allowed limit [2]. In order to avoid damage of the cable near the warning sphere clamps, attention should be paid to determining the optimal position of Stockbridge dampers near the suspension clamps, as well as near the warning sphere clamps. Because of this, a reliable and numerically efficient procedure for the calculation of bending strains at an arbitrary point in the span is needed.

Aeolian vibrations of overhead transmission lines are in practice most often estimated by using the Energy Balance Method (EBM) [9–11,17] and the Modified Energy Balance Method using eigenfunctions (MEBM) [12,26–28]. By applying the EBM, a relatively dense discrete spectrum of natural frequencies of the considered cable is approximated by the continuous spectrum of a semi-infinite one. The energy balance between the power injected into the system through the aerodynamic forces, the power dissipated by the vibration dampers, and the power dissipated by the conductor due to the conductor's self-damping is then carried out for all frequencies of interest. After the average free-field vibration amplitudes are obtained, the bending strains can be determined by using approximate expressions [9]. It must be emphasized that this simple and numerically very efficient method can be applied only if the in-line fittings (e.g., Stockbridge dampers) are spaced near the suspension clamps. If this is not the case, then the MEBM should be used.

MEBM enables determination of the eigenvalues of a cable with in-span fittings (Stockbridge dampers, aircraft warning spheres), and therefore, enables calculation of bending strains at any point in the span. However, the continuous spectrum approximation can no longer be used, and discrete natural frequencies and the corresponding mode shapes of the cable must be determined. As a result, MEBM is numerically a much more demanding procedure than EMB. MEBM requires a sort of optimization procedure to determine the complex frequency parameters that make the system matrix numerically singular. In each step the system matrix is a bit "more singular" (its smallest singular value becomes even smaller) than the matrix in the previous step, i.e., it has a higher condition number. Thus, the first and the very crucial step of applying the MEBM is the accurate determination of the system eigenvalues.

In this paper it is shown that there are certain difficulties in predicting the bending strains in the cable by using MEBM because they considerably depend on the precision of the computed eigenvalues and eigenvectors. The work presented suggests that the procedure of numerical solution of the eigenvalue problem described in [26,28,27], can be significantly improved. The improvement is deemed necessary for two reasons. First, as shown in this paper, the method described in [26–28] can be insufficiently accurate which may lead to an inaccurate determination of the corresponding bending strains. Second, the procedure described in [26–28] may lead to asymmetric cable modes even though a symmetric structure is considered. This is not a physically possible result.

It should be emphasized that MEBM is an approximate method. Its accuracy, apart from the numerical aspects of the method, significantly depends on the assumptions that are built into the model, as well as on the input data (wind power input and the mechanical characteristics of the cable and the Stockbridge damper) that are generally difficult to estimate.

The mechanism of forming and shedding of von Kármán vortices is a very complex phenomenon. The empirical data on the influence of the wind force on the cable, as well as on the power which in that way enters the system differ significantly from source to source in the available literature [15]. The data on the characteristics of air flows (velocity and direction of the airflow, turbulence in the fluid flow) on any given location are typically unknown [15]. It is also difficult to estimate the mechanical characteristics of the cable (bending stiffness, characteristic bending diameter, and self-damping) because they depend on many parameters (cable construction, tension force, vibration frequency and amplitudes, and temperature) [5]. Similar to the wind power input, these data vary considerably from source to source in the literature [4].

Dynamic properties of Stockbridge dampers are usually determined in the lab. The force on the damper clamp is measured while the damper clamp is subjected to controlled translational (harmonic) motion [14]. The frequency response function between the velocity amplitude of the damper clamp and the force acting on it is called impedance. However, for a more accurate description of the damper dynamic properties it is necessary to take into account the rotational motion of the clamp, since both motions co-exist in reality. In this case a Stockbridge damper should be characterized by an impedance matrix connecting forces and torques due to translational and rotational motion of the clamp. This approach is very rarely used in practice due to its complexity. Also, in practice it is usually neglected that the Stockbridge damper impedance is a function of both the frequency and the clamp vibration velocity amplitude, e.g., the nonlinearity of the Stockbridge damper is neglected.

In this study it is shown how to solve the problem of the determination of the natural frequencies by using a hybrid optimization algorithm by either singular value decomposition (SVD), or pivoted QR factorization of the given system matrix. The way of forming the system matrix and the energy balance procedure [26–28] are described briefly in Sections 2 and 5, respectively. A detailed description of some specific and useful properties of the system matrix as well as the procedure of the numerical solution of the eigenvalue problem used in this paper, are given in Sections 3 and 4. Numerical examples presented in Section 6 illustrate advantages of the numerical procedures developed in this paper.

2. Vibration model of overhead transmission line with in-span fittings

The model of an overhead transmission line cable with Stockbridge vibration dampers and aircraft warning spheres is shown in Fig. 2.1.

Overhead transmission line cables are often modeled as beams with bending stiffness EI and tensile forces T at the ends [6,10,11]. Transverse vibrations of each sub-span can be described by a non-homogeneous, non-linear partial differential equation of the fourth order (primes denote the differentiation with respect to the space coordinate x, while dots denote the differentiation with respect to the time t)

$$\begin{aligned} EI\, w_j''''(x_j,\, t) - T w_j''(x_j,\, t) + \rho A\, \ddot{w}_j(x_j,\, t) \\ = q(x_j,\, t) + d_K(w_j,\, \dot{w}_j,\, t), \end{aligned} \tag{2.1}$$

where w_j is the transverse vertical displacement of the cable at a location x_j at time t, EI is the cable bending stiffness, T is the cable tension force, ρA is the cable mass per unit length, $q(x_j,t)$ is the wind force imparted on the cable due to von Kármán vortex shedding, and $d_K(w_j,\dot{w}_j,t)$ is the member representing the cable self-damping.

The transverse string vibration model is also often used since the bending stiffness of the cable is small and its influence on the natural

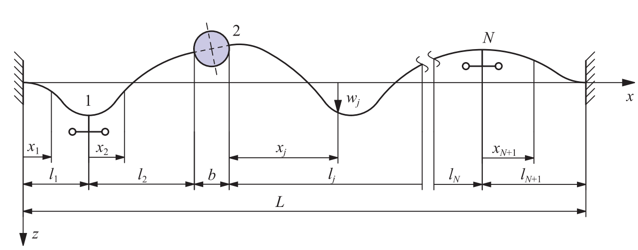

Fig. 2.1. Model of an overhead transmission line cable with Stockbridge dampers and aircraft warning spheres.