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Stability analysis of bi-directionally corrugated steel plates with orthotropic plate model

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ABSTRACT

The aim of this paper is the investigation focused on sinusoidally corrugated in both direction steel plates. The analyzed plates can be used as corrugated webs in cold-formed steel profiles, and then eliminate the need for transverse stiffeners. The stability analysis of the bi-directionally corrugated plates under compression and shear requires the use of shell model. In case of dense corrugation, that model is described by differential equations with strongly oscillating coefficients. The use of such equations in the analysis of engineering issues is very troublesome, even when using finite element method. This work presents one of the averaging procedures leading to equations with constant coefficients, in which the corrugated plates are replaced by a certain equivalent continuous structure with mean stiffness modules that describe the properties of the actual plate. The aim of the study is to demonstrate the possibility of using an orthotropic plate model for the analysis of the stability of this type of steel panels. The averaging effective modules for the orthotropic model has been identified. The proposed model was validated by comparing the critical force values obtained from the orthotropic plate model and the FEM shell model. The proposed procedures only takes into account linear elastic behavior of the material.

1. Introduction

Corrugated plates are used as a component of bridge girders as well as a webs of I-profile beams for many years. The shear stiffness of such webs is higher than stiffness of flat steel plates and eliminates the need for transverse stiffening ribs. The analysis of the stability of corrugated webs is described in many works. Let's mention same of them here: Sayed-Ahmed [1] analyzed the local and global stability of I-beam webs with trapezoidal corrugation. In paper [2], Eldib analyzed the shear stability of bridge girders, where the webs with trapezoidal or sinusoidal corrugation were shaped. Hassanein and Kharoob [3] investigated the influence of real boundary conditions resulting from the web-to-flange connection on the stability of corrugated webs. Unfortunately, the webs with trapezoidal or sinusoidal corrugation besides the advantages also have same disadvantages, such as: technological difficulties with web-to-flange weld connection and susceptibility of the web panel to accordion effect.

The aim of this paper is the investigation focused on sinusoidally corrugated plates in both direction (Fig. 1.1). The corrugation in two direction allows to find a straight line along which the weld connection with other plates can be made, makes the possibility to use ribs, and probably causes that accordion effect does not occur. Due to that properties, it is possible to use such plates not only as a webs of hot-rolled I-beams but also in cold-formed profiles. The literature on the study of bi-directionally corrugated elements, in contrast to unidirectional corrugation, is not very comprehensive. Barholome and Pasternak in [4] and Ossenbrink et al. in [5] presents the results of tests using structured steel panels shown in Fig. 1.2. The paper [4] is focused on the experimental research and numerical analysis of the stability of various web panels, including structured webs with three types of corrugations.

In this paper the stability of the bi-directionally corrugated plates under compression and shear will be analyzed. The analysis of stability of one-direction and bi-directional corrugated panel requires the use of shell model. In case of dense corrugation, that model is described by differential equations with strongly oscillating coefficients. The use of such equations in the analysis of engineering issues is very troublesome, even when using finite element method. Hence, there is the need to search for certain substitute models described by the averaged equilibrium equations. The equations of these averaged models are described by constant coefficients of the effective modules in contrast to shell model equations with strongly oscillating coefficients. This work presents one of the averaging procedures leading to equations with constant coefficients, in which the corrugated plates are replaced by a

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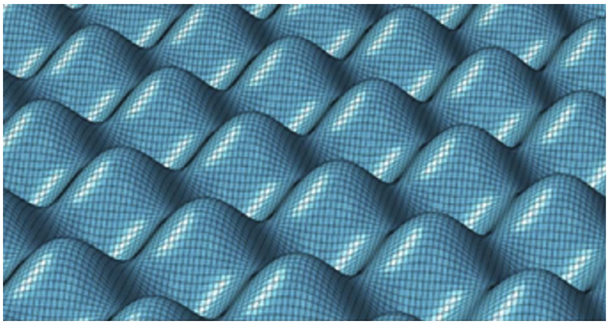

Fig. 1.1. Bi-directionally corrugated plate.

certain equivalent continuous structure with mean stiffness modules that describe the properties of the actual plate. The aim of the study is to demonstrate the possibility of using a orthotropic plate model for the analysis of the stability of this type of steel panels. The averaging effective modules for the orthotropic model has been identified. The proposed model was validated by comparing the critical force values obtained from the orthotropic plate model and the FEM shell model.

2. Macroscopic model of orthotropic plate

The object of our considerations is a rectangular plate with a periodic structure in two direction (Fig. 2.1) with periodic cell shown in Fig. 3.1. Let us introduce the Cartesian coordinate system $0x_1x_2x_3$. The central surface of the considered plate is described by Eq. (2.1):

$$x_3(x_1,x_2) = z(x_1,x_2) = f\cdot\sin\left(\frac{m\cdot\pi\cdot x_1}{a}\right)\cdot\sin\left(\frac{n\cdot\pi\cdot x_2}{b}\right) \tag{2.1}$$

where

f-amplitude of corrugation,
m-number of waves in x_1 direction,
span of plate in x_1 direction,
n-number of waves in x_2 direction,
span of plate in x_2 direction.

Plates with thickness $t = 0.4; 0.6; 0.8; 1.0; 1.5; 2.0; 2.5; 3.0$ mm and with the amplitude of corrugation equal to $f = 0; 1; 2; 3; 4; 5; 6; 7$ mm were analyzed.

The periodic cell of the plate shown in Fig. 3.1 has dimensions equal to $l_1 = l_2 = l = 0.20$ m, and its central surface is described by Eq. (2.2):

$$z(x_1,x_2) = \sin\left(\frac{2\cdot\pi\cdot x_1}{20}\right)\cdot\sin\left(\frac{2\cdot\pi\cdot x_2}{20}\right) \tag{2.2}$$

The stability analysis of corrugated plates from shell models is too complicated for engineering applications. Hence the need to search for simplified models. One of the most common one is the orthotropic plate model (Troitsky [6]). The stability of a rectangular orthotropic plate describes the known Eq. (2.3):

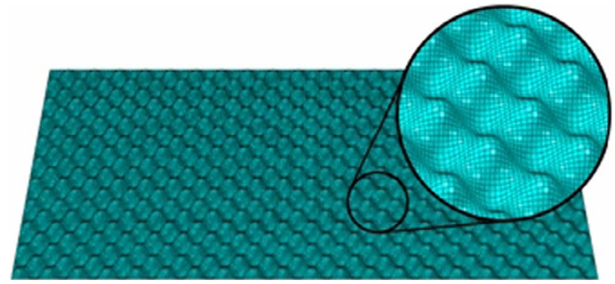

Fig. 2.1. An example of analyzed plate.

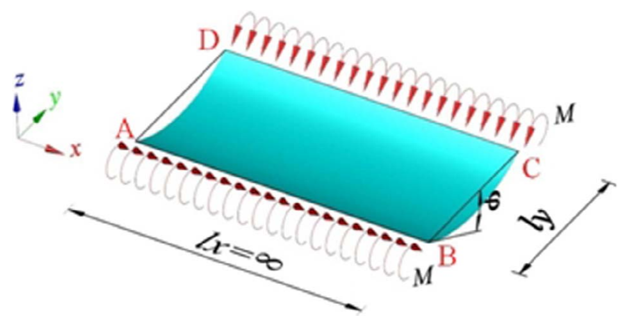

Fig. 2.2. The D_{11} stiffness determination scheme.

Table 2.1
Boundary condition for D_{11} stiffness determination.

	Symbol	Edge						Node			
		A-B		B-C	C-D		D-A	A	B	C	D
Displacement	δ_x	–	U	–	–	U	–	R	–	–	–
	δ_y	–	U	–	–	U	–	R	R	–	–
	δ_z	R	U	–	R	U	–	R	R	R	R
Rotation	θ_x	–	U	–	–	U	–	–	–	–	–
	θ_y	–	U	–	–	U	–	–	–	–	–
	θ_z	–	U	–	–	U	–	–	–	–	–

where: R – restrained, U – deformation dependency of all nodes of a given edge from one another.

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2H\frac{\partial^4 w}{\partial x^2\partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = P-\left(N_{11}\frac{\partial^2 w}{\partial x^2} + 2N_{12}\frac{\partial^2 w}{\partial x\partial y} + N_{22}\frac{\partial^2 w}{\partial y^2}\right) \tag{2.3}$$

where

D_{11}, D_{22} – bending stiffness of orthotropic plate,
$H = D_{11} + 2D_{66}$ – effective torsion stiffness of orthotropic plate,
D_{12} – shear stiffness,

Fig. 1.2. Types of structured steel plates analyzed in [4,5].