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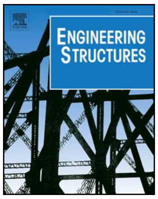

Stocky thin- or thick-walled beams: Theory and analysis

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ABSTRACT

The *nodal-line method* (NLM) is proposed for treating the wide-flange stocky thin- or thick-walled beams featured by (1) clear longitudinal axis, (2) low length/width ratio ($\leq$ 3), and (3) three beamlike stress components. The nodal lines parallel to the axis are distributed on all sides (for both thin and thick walls) plus the interior (for thick walls only) of the beam, and used as the reference frame for imposing the 3D displacement field. The axial and transverse displacements of the nodal lines are taken as the unknown functions and used along with interpolation functions to describe the displacement field. By the principle of minimum potential energy, a set of ordinary differential equations (ODE) and boundary conditions are established for the beam, which are solved by existing ODE solvers. The displacements and stresses of the beam so computed can duly account for the shear-lag effect of wide-flange box beams. For long and medium-long beams, the stocky beam reduces to the Bernoulli-Euler or Timoshenko beam, depending on the range of slenderness ratios. Either asymmetric bending, restrained torsion, or cross-sectional warping of box girders can be easily treated. More phenomena will be explored in the exemplar study of various box girders.

1. Introduction

Research on box girders has long been a subject of high interest to structural analysts, because of their popular application in bridges and viaducts. Recently, it has grown to an unprecedented level due to the booming construction of high-speed railways and other transportation networks in the world, specifically in China and countries located along the Silk Road Economic Belt [1,2]. However, the increasing use of wide-flange thick-walled box girders has brought three challenges to the classical beam theory developed by scientists including Bernoulli-Euler [3], St. Venant [4], Timoshenko [5], and Vlasov [6], or the extended or improved beam theory [7,8].

The first challenge is related to the *shear lag effect* in wide-flange box girders, even though some excellent researches have been carried out along these lines [9–14]. The question is whether or not the flange plate (cantilevered slab) with varying thickness of a box-girder affects the distribution of the normal stress on the flange plate itself? In other words, in transition from a thick- to thin-walled cross section, how will the phenomenon of shear lag affect the normal stress distribution of each flange?

The second challenge is the *restrained-torsional problem* of a wide-flange thick-walled box girder. It is quite often that a thick-walled haunched trapezoidal cross section with two wide flange slabs is adopted for the bridge girder so as to broaden the deck supporting area (Fig. 1). The asymmetry of the cross section makes the shear centre (for specific cases) or twist centre (for general case)[1] different from the centroid, leading to the fact that the position of the shear centre or the twist centre cannot be easily determined by the conventional method [16]. Thus, the problem of restrained-torsion cannot be solved using the conventional beam theory, which will be encountered in cases involving both bending in the asymmetric (x-y) plane (Fig. 2) [17,18] and torsion about the shear centre of wide-flange thick-walled box girders [19].

For the design of modern box girders, we need to know the behavior of bending of the box girder in the asymmetric (x-y) plane, rotation of the cross section around the twist centre, and the coupling between bending and torsion when subjected to loads not passing through the twist centre. Recently, box girders have been popularly used as the viaducts in China, while numerous bridges with piers over 50 m (about 17-storey building high) have been built in various parts of the country. For such bridges, both the effects of lateral winds and seismic forces should be considered in design. Since the resulting lateral force does not

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[1] Andreaus and Ruta pointed out that the shear centre and twist centre differ for a wide range of cross sections, and they only coincide for some specific cases [15]. Only the twist centre is of concern herein as we are discussing the general case of thin- and thick-walled sections.

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Fig. 1. A thick-walled haunched trapezoidal cross section with wide flange slabs.

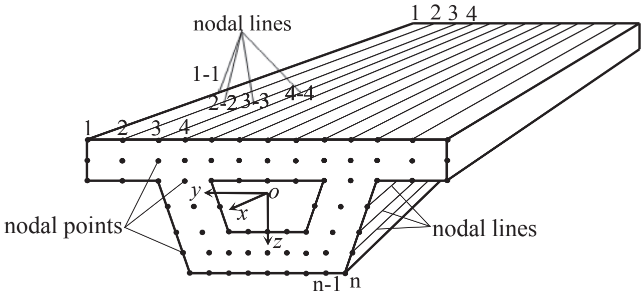

Fig. 2. Nodal lines and nodal points of the box girder.

pass through the shear centre of the cross section, the box girder will be bent in the asymmetric (*x*-*y*) plane (Fig. 2) and twisted around the minimum torsional stiffness centre, due to the bending-torsional coupling [17,18].

The third challenge is that the *length to width ratio of less than 3* for bending in the asymmetric (*x*-*y*) plane of the box girder (Fig. 2) is beyond the applicability of existing beam theories. In analyzing the bending of a box girder in its asymmetric plane, the span-to-width ratio, rather than the span-to-height ratio, should be adopted. In practice, there exists quite a number of box girder bridges with a span/width ratio of less than 3, e.g., the box girder used in the Wuhan light rail has a length of 25 m, height of 1.946 m, and a flange width of 8.7 m, implying a span/width ratio of 2.87 [20]. For this type of bridges, the following beam theories with certain hypothesis for out-of-plane cross-sectional displacements cannot be directly applied: the Bernoulli-Euler beam theory (with plane-section hypothesis), St. Venant's beam theory (with free warping out of the plane of the cross section), Vlasov's thin-walled beam theory (with zero shear strain in the midline of an open cross section, or constant shear flow around the midline of a closed cross section), and other extended or improved beam theories [7,8].

Partly to overcome the above challenges, an abundance of research works has been carried out on box girders. Nearly 400 publications associated with the shear lag problem of box girders can be found in the single source of the SD Elsevier database. Readers who are interested in the shear-lag research of box girders should be referred to the early work by Reissner [9] and to Refs. [10–14] for a historical review. There exist numerous researches on the *bending* and *torsional* behaviors of box girders. The following are the recent pertinent publications available in the SD Elsevier database.

Choi et al. [21] analyzed the deflections of prestressed concrete (PC) box-girder bridges using the spline finite strip method with the non-periodic B-spline interpolation. Wang et al. [22] presented an efficient finite segment method for the analysis of curved box girders with corner stiffeners, ignoring the deformation along the width or depth of the plates of the cross section. Hii and Al-Mahaidi [23] reported an experimental and numerical study on torsional strengthening of solid and rectangular box reinforced concrete (RC) beams with externally-bonded carbon fibre reinforced polymer. Galal and Yang [24] performed an experimental and analytical study on the effect of bottom slabs on the ultimate loading capacity of haunched thin-walled RC girders under centric and eccentric monotonic loads. Ramosa et al. [25] studied some unusual secondary structural effects in a variable-depth box girder bridge, such as transverse axial forces in the webs, transverse bending in the bottom slab, longitudinal bending in the webs, horizontal bending from shear lag, and global versus local bending. Dowell and Johnson [26] presented a closed-form approach for determining the torsional constant and shear flows for multi-cell cross sections under torsion. Based on the test results from 152 specimens, Rahal [27] proposed a simple non-iterative approach for calculating the ultimate torsional moment in normal- and high-strength RC beams.

Regarding the *bending and torsional coupling* of beams, there also exists a number of studies in the literature, of which only a few are cited here. Initially, Timoshenko and Young [28] began to study the bending-torsional coupling vibration of beams with circular and square cross sections ignoring the cross-sectional warping. Subsequently, the study of coupled bending and torsional vibration was extended to other beams in the works by Dokumaci [29] and Li et al. [30].

As for box girders, there exists quite a number of researches. Taysi and Ozakca [31] used the finite strip method to study the free vibration of thin-walled box girders in straight and curved platform. Ashebo et al. [32] measured the static and dynamic bending moments of a skew box-girder continuous bridge induced by a three-axle heavy truck, and concluded that the skew effect is small for skew angles less than 30°. Ramanathan et al. [33] presented the temporal evolution of seismic fragility curves for the design of concrete box-girder bridges in California based on a review of the design details. Moon et al. [34] proposed approximate methods for locating the shear centre, calculating the warping constant of I-girders with corrugated webs, and determining the lateral–torsional buckling strength under uniform bending. Using full-scale models, Chung and Kim [35] compared the dynamic properties of spliced and monolithic precast PC box railway bridges, showing that the dynamic performance of the spliced girder is comparable to the monolithic girder in both uncracked and cracked states. Zaine et al. [36] analyzed the free vibration of thin- and thick-walled box beams made of functionally graded materials using the first-order shear deformation theory (FSDT), in which the primary and secondary torsional warping, shear and bending deformations were incorporated. Ovesy and Masjedi [37] investigated the effect of constitutive equations on the free vibration of single-celled thin-walled composite box beams, in which some non-classical effects such as restrained warping and transverse shear were considered. Yang et al. [38] performed wind tunnel tests to study the aerodynamic instability of twin box girders for long-span bridges. Zhang et al. [39] conducted a field measurement for the interior low frequency noise from a simply-supported box-shaped bridge induced by running trains. Abid et al. [40] experimentally analyzed the temperature distributions in concrete box girders, and proposed empirical formulas for predicting the maximum temperature gradients and mean temperature of the bridge.

The above survey of literature on box girder research reveals that the aforementioned three challenges to the existing beam theories remain basically unresolved. For box girders with a length/width ratio of less than 3, the above challenges can be interpreted as: (1) To determine the location of the twist centre through which the cross section of the box girder rotates in practice; (2) to calculate the cross-sectional warping induced by restrained torsion; (3) to develop a more effective approach (than most existing numerical ones) for modeling the coupling bending and torsion for loads not passing through the twist centre.

The present paper is featured by the following facts: First, since no assumption is made for the wall thickness of the cross section, the equation presented for determining the twist centre works for all types