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# On the assumed inherent stability of semi-active control systems

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A R T I C L E I N F O

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A B S T R A C T

Vibration control systems are usually classified into: passive, active and semi-active. Semi-active control systems are based on formerly passive mechanical devices, such as springs and dampers, whose characteristics are adjusted in real-time by active means. The attractiveness of semi-active control systems mainly relies on their assumed “inherent stability”, which makes them almost as reliable and fault-tolerant as passive control systems.

The present paper shows that these assumptions are only partially true, by applying passivity formalism and bounded-input bounded-output stability definitions. Based on this study, semi-active control devices are rationally classified into three classes with two subclasses each: (1.1) non-negative variable-damping dampers, (1.2) possibly-negative variable-damping dampers, (2.1) independently-variable-stiffness springs, (2.2) resettable-stiffness springs, (3.1) independently-variable-inertance inerters, and (3.2) resettable-inertance inerters. It is found that a control system using any of the semi-active control devices of type (1.2), (2.1) or (3.1) is not inherently stable, as it is assumed in some previous papers; because those devices are “active” from the perspective of the passivity formalism. Interestingly, hybrid combinations of independently-variable-inertance inerters with non-negative variable-damping dampers can be designed to produce inherently-stable control systems. Following this framework, several published works on semi-active control systems are reviewed and classified.

The presented methodology is useful when developing new devices. This is demonstrated by proposing a novel control device, which is classified and assessed in terms of inherent passivity. Moreover, this passivity assessment is conveniently used to propose a control law for the device. Finally, a frame structure controlled by the device is numerically simulated through a number of scenarios including instability and a countermeasure for its mitigation.

## 1. Introduction

Structural vibrations are detrimental to the performance of many engineering applications and, therefore, several methods of vibration control have been proposed to reduce them. These methods are generally classified into: Passive Control (PC) [1,2], Active Control (AC) [3,4], or Semi-Active Control (SAC) [5,6]; although hybrid combinations (HC) are also common [3]. In a SAC system, the properties of *formerly* passive devices (e.g., viscous dampers, springs, pendula) are conveniently adjusted in real-time through auxiliary actuators (e.g., valves, motors) according to a control law [7]. SAC is attractive since it offers the reliability of PC; while approaching the adaptability and effectiveness of AC, without imposing high power demands. Furthermore, it is common to assume that SAC systems are “inherently stable” [6].

An important benefit of “inherent stability” is a guaranteed stability irrespectively of control-law design, modelling errors, and failures in miscellaneous hardware of the SAC system, i.e. sensors, transmission equipment, control computers, auxiliary actuators, and power supplies (see Fig. 1). The fault-tolerance of these subsystems is particularly important in three cases: (1) applications that remain in standby for many years until their operation is needed, as mitigation of seismic vulnerability in civil structures [8]; (2) applications under harsh environments, such as smart suspension systems for vehicles [9]; and (3) applications deployed in remote locations, such as artificial satellites and other space structures [10]. AC systems, which are not inherently stable, can yield a dangerously large structural response if any fault is present in the hardware or if its design is inadequate.

An additional benefit of “inherent stability” is that researchers and practicing engineers that are not familiar with non-linear control theory can use and/or propose new semi-active devices without destabilization risk. Moreover, new control laws can be proposed without stability analysis; e.g. heuristically as in [11]. This advantage of the assumed “inherent stability” is important since vibration control design is a multidisciplinary task that involves not only control engineers but also

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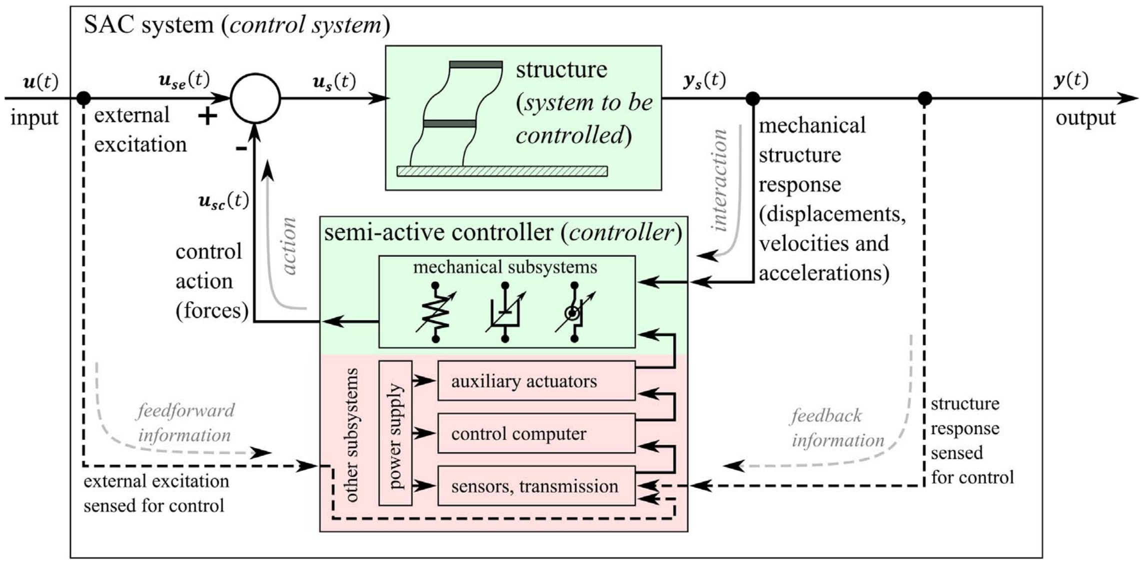


**Fig. 1.** General semi-active control system.

mechanical, electronics and civil engineers.

Many investigations [6,8,12–20] appeal to the "inherent stability" of SAC systems and justify it on the "passivity" of the devices used to implement them; although it has also been claimed that "inherent stability" cannot be generalized [21,22]. This inconsistency arises from the use of ambiguous definitions. Since their introduction in the 1970s [7], many new semi-active devices have been proposed; e.g. variable-damping dampers [7], variable-stiffness springs [23] and, recently, variable-inertance inerters [24,25]. As a consequence, the definition of SAC is often adapted in order to encompass the new devices, which leads to an increasing risk of misunderstanding.

The purpose of this paper is to clarify the definitions of "semi-active", "stability", and "inherent stability", in order to formally address the issue of the "inherent stability of semi-active control systems" within a general framework. The approach proposed is based on the passivity formalism [26], as suggested by Hrovat [27] for classifying "active" and "passive" suspension systems. Among the many available definitions of "stability" [28], this study considers bounded-input bounded-output (BIBO), which can be deduced from the passivity theorem [29] and is appropriate for systems under forced excitation. Finally, this paper denotes the stability as "inherent" when it depends exclusively on the mechanical subsystems (the green blocks in Fig. 1) of the controller.

## 2. Definitions and nomenclature

### 2.1. Mathematical preliminaries

In order to establish the boundedness of vector-valued functions, such as displacements, velocities, accelerations and forces, the $L_2$ norm of a vector function $x$ is defined as:

$$||\boldsymbol{x}||_2 = \sqrt{\langle \boldsymbol{x},\boldsymbol{x} \rangle} \tag{1}$$

where $\langle \cdot,\cdot \rangle$, the inner product of vector functions, is defined as:

$$\langle \boldsymbol{x},\boldsymbol{y} \rangle = \int_0^\infty \boldsymbol{x}'(t)\boldsymbol{y}(t)dt \tag{2}$$

where $'$ is the transpose operator. Note that $||x||_2$ is a scalar denoting the norm of the vector-valued function $x$; which must not be confused with $||x(t)||_2 = \sqrt{x'(t)x(t)}$.

Similarly, the truncated inner product of $x$ and $y$ over the interval $[0,T]$, is defined as [30]:

$$\langle \boldsymbol{x},\boldsymbol{y} \rangle_T = \int_0^T \boldsymbol{x}'(t)\boldsymbol{y}(t)dt \tag{3}$$

where $T$ is a particular instant of time; and the truncated $L_2$ norm of $x$ as follows [30]:

$$||\boldsymbol{x}||_{2,T} = \sqrt{\langle \boldsymbol{x},\boldsymbol{x} \rangle_T}. \tag{4}$$

Thus, a function $x$ lies in the $L_2$ space, i.e. $x \in L_2$, if $||x||_2 < \infty$ [28]. Correspondingly, a function $x$ lies in the extended- $L_2$ space, i.e. $x \in L_{2e}$, if $||x||_{2,T} < \infty \; \forall \; T$ [28].

Below, some useful inequalities are summarized [26,31]:

$$\lambda_{\min}(\boldsymbol{A})\boldsymbol{x}'(t)\boldsymbol{x}(t) \leqslant \boldsymbol{x}'(t)\boldsymbol{A}\boldsymbol{x}(t) \leqslant \lambda_{\max}(\boldsymbol{A})\boldsymbol{x}'(t)\boldsymbol{x}(t). \tag{5}$$

$$\langle \boldsymbol{x},\boldsymbol{y} \rangle_T \leqslant ||\boldsymbol{x}||_{2,T}\,||\boldsymbol{y}||_{2,T}. \tag{6}$$

$$||\boldsymbol{x}+\boldsymbol{y}||_{2,T} \leqslant ||\boldsymbol{x}||_{2,T} + ||\boldsymbol{y}||_{2,T}. \tag{7}$$

where $\lambda_{\min}(\boldsymbol{A})$ and $\lambda_{\max}(\boldsymbol{A})$ are the smallest and largest eigenvalues of $\boldsymbol{A}$, Eq. (6) is the Cauchy–Bunyakovsky–Schwarz (CBS) inequality, and Eq. (7) is the triangle inequality.

From the perspective of vibration control engineering, $L_2$ norm ($||\cdot||_2$) measures a response function in *RMS sense*, which is an evaluation criteria commonly used in that field [32]. Other important criterion is the measurement of responses in a *peak sense*. In this regard, the $L_\infty$ norm and its truncated version are defined as:

$$||\boldsymbol{x}||_\infty = \sup_{t\in[0,\infty)} |\boldsymbol{x}(t)| \tag{8}$$

$$||\boldsymbol{x}||_{\infty,T} = \sup_{t\in[0,T]} |\boldsymbol{x}(t)| \tag{9}$$

Thus, a function $\boldsymbol{x}$ lies in the $L_\infty$ space, i.e. $\boldsymbol{x} \in L_\infty$, if $||\boldsymbol{x}||_\infty < \infty$ [28]. Correspondingly, a function $\boldsymbol{x}$ lies in the extended-$L_\infty$ space, i.e. $\boldsymbol{x} \in L_{\infty e}$, if $||\boldsymbol{x}||_{\infty,T} < \infty \; \forall \; T$ [28].

### 2.2. Definition of "stable"

A system is BIBO stable when the norm of the system output is finite for any input with finite norm. Formally, a system whose input is $\boldsymbol{u}$ and output is $\boldsymbol{y}$ is $L_2$-stable if [28]:

$$\boldsymbol{u} \in L_2 \Rightarrow \boldsymbol{y} \in L_2. \tag{10}$$

Moreover, if there exists a finite constant $\gamma > 0$ such that:

$$\boldsymbol{u} \in L_2 \Rightarrow ||\boldsymbol{y}||_2 \leqslant \gamma ||\boldsymbol{u}||_2, \tag{11}$$

The system is said to be $L_2$-stable with finite gain ($\gamma$) and zero bias.