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Progressive instability in circular masonry columns

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ABSTRACT

The instability behaviour of eccentrically loaded circular masonry columns is investigated. Two approaches are considered for the analysis. One is based on a semi-analytical formulation of the relevant boundary-value problem for a no-tension material response; the other employs a plastic-damage-contact constitutive model, the CraftS model, to capture the complex microstructural behaviour of the material. The latter has been implemented in the finite element program LUSAS and has been already successfully employed to describe progressive instability in eccentrically loaded brickwork wallettes of rectangular cross section. Equilibrium paths and limit load estimates are computed for both analysis approaches for a range of column aspect ratios and load eccentricities. It is shown that the type of material response becomes less important for specimens with height-to-diameter aspect ratios greater than 7.5 and for loads applied to points in the kernel of the cross section, while for higher eccentricities the presence of a tensile strength increases considerably the limit load. The damage evolution predicted by the models is also investigated for selected cases, showing that the formulation based on the no-tension material is able to capture with good agreement the damaged zone of the column for loads with low eccentricities. For the same type of loading, a useful design formula is provided.

1. Introduction

The mechanical response of axially compressed brick masonry walls and columns is greatly affected by the eccentricity of the load. This dependency was well-known to ancient builders and has been investigated experimentally by a number of researchers [1–6]. As far as modelling is concerned, this problem can be tackled by assuming different hypotheses formulated to allow for the main features of masonry whose response is strongly nonlinear; these features being, for both bricks and mortar, a high resistance in compression, a low tensile strength and elastic-plastic damaging behaviour of the two materials. In addition, friction at their interface may be further considered. These features have been combined in different ways leading to various homogenization constitutive models for brickwork [7–11].

When a continuous beam-column model is required, the assumption that the material is elastic in compression and without any resistance in tension (the so called *no-tension material* model) can be postulated. In the past, many researchers have used this assumption to estimate failure modes and loads of a range of compressed slender masonry structures. In particular, Sahlin [12], Yokel [13] and Frisch-Fay [14] were the first to investigate progressive instability, and maximum loads, in non-centred loaded pillars. Their approach was subsequently extended by the addition of self-weight effects (see Refs. [15,16]). The continuous assumption, which yields second-order differential governing equations to be integrated with the relevant boundary conditions, was then followed by the development of a 'discrete' approach [17] in which the pillar was divided into a finite number of blocks (or elements). The associated algebraic equations were formulated by imposing appropriate equilibrium conditions for the individual elements. The instability of homogeneous eccentrically loaded masonry pillars has been also investigated numerically, mainly using the Finite Element (FE) method. Ganduscio and Romano [18] developed a FE technique to consider no-tension materials with a non nonlinear response in compression. Brencich and Gambarotta [19] and Adam et al. [20] adapted a nonlinear plastic-damage-contact material model, originally developed for concrete, to successfully describe the behaviour of masonry wallettes.

The majority of previous analyses considered prismatic specimens of rectangular cross section, although there have been a few previous investigations on the instability of circular cylinders [21,22]. In addition, Gurel [23] recently presented a study in which the load-bearing capacity of eccentrically loaded slender no-tension circular columns (including self-weight effects) was assessed with the discretised model proposed in [17]. In his paper, unbounded linear elastic compression was assumed and specimens with height-to-diameter ratios greater than 12.5 were considered. Moreover, results were compared with those

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obtained with a commercial FE package adopting the same constitutive model.

Based on the above introduction and state-of-the-art review, this paper describes four main contributions to the topic, as follows:

(i) a method for determining the beam-column governing equations necessary to compute the compressive load-lateral displacement curve of a circular column comprising no-tension material loaded with a small or a large eccentricity, along with a numerical technique for solving these equations;
(ii) the application of the above model to a set of circular columns loaded at various eccentricities, along with the computed equilibrium paths, limit loads, and the extent of the cracked zones to enhance the dataset provided in [23];
(iii) the application of a fully coupled plastic-damage-contact (CraftS) constitutive model suitable for brickwork, implemented in a FE package, to the evaluation of the performance of the same specimens analysed in ii) so that an incisive comparison between different approaches can be made;
(iv) a new formula which captures, with a good agreement, the limit load vs eccentricity behaviour of slender and relatively-slender columns, formed of a zero tensile strength material.

The paper is divided into six Sections. After the introduction, in Section 2, the constitutive models adopted in the analysis are introduced. In Section 3, the method to solve the analytical formulation of the beam-column boundary-value problems is illustrated, while the general setting for FE simulations is described in Section 4. Results and comparisons are contained in Section 5, followed by concluding remarks.

2. Models for the eccentrically compressed circular column

The features of the models considered in the paper are described in this Section. The prismatic column has diameter D and height L and x denotes the axial coordinate. The structure is assumed to be clamped at cross section $x = 0$ and free at the top ($x = L$), where the resultant compressive load P is applied with the given eccentricity e (Fig. 1a).

2.1. No-tension material with linear elastic compressive branch

A no-tension (NT) material is unable to withstand any tensile stress, while compression can be described in different ways, according to the nature of the solid. In our study, only vertical displacements at the base are constrained. Here, it is assumed that the uniaxial compression branch is unbounded linear elastic (E is Young's modulus). This hypothesis has led to closed-form solutions of the progressive instability of pillars with rectangular cross sections [12–14,24,25]. For a circular cross section, as shown later, an analytical solution is not available, but the resulting second order ODE can be readily numerically integrated. Although the NT model can be seen as the natural candidate for dry-joint masonry, its use for clay/mortar brickwork columns will be later assessed.

2.2. Plastic-Damage-Contact constitutive model (CraftS)

3D finite element simulations of the loaded column were carried out with the package LUSAS (ver 15.2) using a plastic-damage-contact constitutive model (named the CraftS model, originally formulated for concrete in [26]). An earlier version of this model was successfully employed to assess the behaviour of eccentrically loaded clay brickwork wallettes [20]. The model is able to simulate directional cracking, crack closure and the effects of frictional behaviour in compression, including triaxial confinement [27,28]. CraftS uses a crack-plane sub-model, which is based on a damage-contact formulation, and then incorporates the associated inelastic strains -derived from this sub-model- into a 3D elasto-plastic framework.

The triaxial plasticity component of the model employs a smooth triaxial yield surface due to Lubliner et al. [29], which is rounded using Willam and Warnke's [30] smoothing function. A work hardening hypothesis is used to simulate friction hardening behaviour in a manner similar to that used in the plasticity model presented in [31]. The friction hardening function is linked to a uniaxial compression curve for concrete such that the input parameters are the compressive strength (f_c) and strain at peak compression (ε_c). The friction hardening parameter (Z) has a range [Z_0, 1], with $Z = Z_0$ defining the initial slope of the yield surface and $Z = 1$ giving its maximum value. Z_0 defines the limit of the initial elastic region and is typically set to a value of 0.6, as explained in Ref. [26]. The other parameters used to govern the shape of the triaxial yield function are f_c and the biaxial strength ratio (b_r) (see [26]). It is noted that b_r is defined as f_b/f_c, with f_b being the maximum compressive stress in a biaxial test when the principal stresses are $\sigma_I = 0$ and $\sigma_{II} = \sigma_{III} = -f_b$, assuming a tension positive convention.

The evolution of directional damage (or cracking) is governed by the following damage evolution equation, which was derived from the uniaxial stress-relative-displacement response of a notched axially loaded concrete specimen:

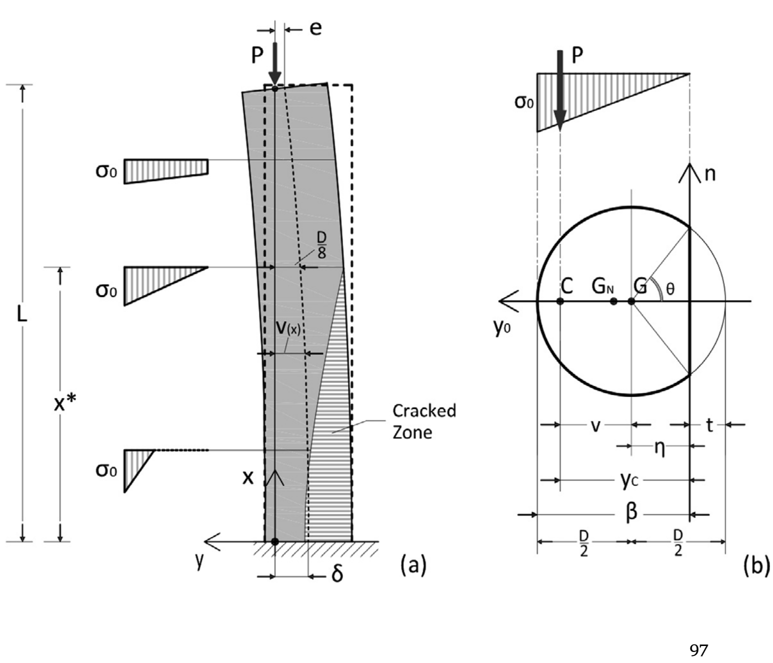

Fig. 1. Geometry and notation of the prismatic circular column: (a) lateral view, where the quantities useful to describe the model based on no-tension material are also reported; (b) cross section and stress distribution in the cracked zone in the case of no-tension material constitutive assumption (G is the centroid of the whole circle while G_N denotes the centroid of the compressed part).