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R-Funicularity of form found shell structures

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A R T I C L E I N F O

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A B S T R A C T

The study of new design methods targeted to minimize the use of materials is a theme of great relevance nowadays; structural designers pursue structural solutions characterized by efficiency, sustainability and optimization. Funicular systems adopt the "right" shape in accordance with the applied load and are ideally able to act without introducing bending. In this work an effective and easy-to-read method to study and quantify the funicularity is presented and applied to structural shells obtained using form finding, and analyzed under different static loads. In order to formulate the new method, the classical funicularity concept has been extended and the definition of *Relaxed Funicularity* (*R-Funicularity*) introduced. The parameter used to define the funicularity is the eccentricity and a structural shell is called *R-Funicular* when the eccentricity is included into an admissibility interval.

1. Introduction

Quoting Sergio Musmeci [1,2]: "There is no reason why the unknown factors should always be the internal stresses and not, for example, the geometric parameters which define the form itself of the structures, since in this latter case a uniformity of stresses and a more complete and efficient use of material may be obtained. With this method, it is possible to arrive at a synthesis of new forms rich in expressive strength." Musmeci's quote is of great relevance to our contemporary societal context, the construction industry is largely responsible for CO_2 emissions [3] and structural designers could reduce this negative impact by targeting efficient structural behavior and use of materials. Among efficient and optimized structural systems, funicular structures adopt the "right" shape in accordance with the applied load. Funicular shell structures are ideally able to resist external loads using membrane forces, mainly tension or compression forces, without introducing bending. As a result their thickness can be minimized and the amount of material reduced. Different form finding techniques have been developed and used to achieve "optimal" structural geometry in static equilibrium with a design loading [4]; for shell structures this loading is usually a gravity load due to the dominance of the shell's self-weight with respect to other applied loads [5].

1.1. About funicularity

The idea of funicularity has been formalized from an analytical point of view between 15th and 17th century. Otherwise one can observe many funicular structures constructed before this time, thanks to experience and static considerations of the designers and constructors; suspended bridges and corbelled domes represent some early examples. During the Roman period, builders seem to have some awareness of funicularity expressed in attempts to change load distributions to achieve better structural stability (e.g. use of filler materials or use of concrete with graded density) [6]. In order to find some written essays concerning this topic, one needs to look at the 13th century. The medieval architect Villard de Honnecourt in his manuscript "Livre de portraiture", between sketches and notes, describes how to construct a cross vault optimizing the entire process with the application of the rule of the three arches [7]. At a later date, it has been confirmed that cross vaults constructed respecting this rule have a better structural behavior since the bending stresses are reduced [8]. This could demonstrate an intuition of the relationship between shape and performance of the structure, even if this concept was not analytically expressed during the Middle Ages [9]. Starting from the 15th century, the first studies on arches and cables appear. Theoretical definitions attempt to justify and formalized what was experimentally evident. Leon Battista Alberti (1404–1472), Andrea Palladio (1508–1580), Leonardo da Vinci (1452–1519) and Simon Stevin (1548–1620) are some of the most celebrated scientists to give a fundamental contribution to the formulation of the behavior of curved structure and of the arch equilibrium [6]. Galileo Galilei (1564–1642) was the first one who attempted to give a mathematical description of a cable; in his writing "Dialogues Concerning Two New Sciences" (1638), mistakenly using an erroneous analogy with the parabolic motion of

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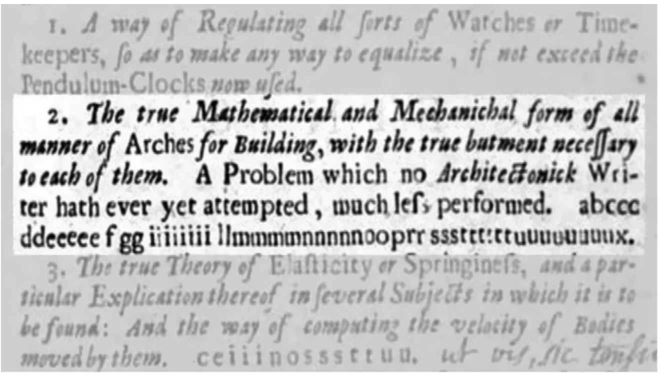
1. A way of Regulating all sorts of Watches or Time-keepers, so as to make any way to equalize, if not exceed the Pendulum-Clocks now used.

2. The true Mathematical and Mechanichal form of all manner of Arches for Building, with the true butment necessary to each of them. A Problem which no Architectonick Writer hath ever yet attempted, much less performed. abcccddeeeeefggiiiiiiiillmmmmnnnnnooprrsssttttttuuuuuuuux.

3. The true Theory of Elasticity or Springiness, and a particular Explication thereof in several Subjects in which it is to be found: And the way of computing the velocity of Bodies moved by them. ceiiinosssttuu.

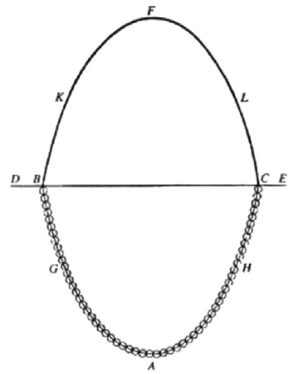

Fig. 1. Robert Hooke's anagram (Hooke, 1676) and Poleni's drawing of Hooke's analogy between an arch and a hanging chain.

projectiles, he confused catenary and parabola [10]. Joachim Jungius (1587–1657) rejected Galilei's statement, demonstrating the difference between the two, which increases when the sag to span ratio decreases. Jungius's writing "Geometria Empyrica" was published in 1669 after his death [6]. The correct equation of a cable's geometry was written in 1691 by Gottfried Wilhelm von Leibniz (1646–1716), Christiaan Huygens (1629–1695) and Johann Bernoulli (1667–1748) [11]. Huygens was the first one to use the term "*catenaria*" in one of his missive to Leibniz. English engineer and scientist Robert Hooke (1635–1703) gave an additional fundamental contribution in 1676 publishing the Latin anagram indicated in Fig. 1. The solution was published by the secretary of the Royal Society, Richard Waller in 1705 and read "Ut pendet continuum flexile, sic stabit contiguum rigidum inversum", the translation is "As hangs the flexible line, so but inverted will stand the rigid arch" [12]. The concept is simple: in order to obtain an arch that acts in pure compression, the shape of the equivalent hanging chain needs to be inverted. During the same years, David Gregory (1659–1708) stated that an arch is stable if the thrust line, that is the line representing the path of the resultants of forces acting in a structure, lies within its thickness [13]. This is the basic concept behind the structural assessment of masonry structures. Some years later, following Gregory's studies, Claude-Louis Navier (1785–1836) and E. Méry (1840) supposed that in order to have an arch fully compressed, the thrust line would have to lie within the middle third of his section [14]. The thrust line becomes an indicator of the stability of arches: the more this line lies away from the axis of the arch, the more its thickness needs to be increased. Hence the "right" shape for an arch is the one corresponding to the funicular of the loads applied.

1.2. Form finding

In the last two centuries, three different groups of methods have been formalized to "find the right shape" and define funicular geometries. The first one uses physical models where bending stiffness is neglected, as hanging chains or membranes or thread models [4]. The inversion principle is applied to find bending-free shapes. Sir Christopher Wren (1632–1723) and Giovanni Poleni (1683–1761) used chain models to design the dome of the St. Paul's Cathedral in London and to assess the structural behavior of the dome of the St. Peter's Cathedral in Rome, respectively. More recently, Antoni Gaudì (1852–1926), Frei Otto (1925, 2015) and Heinz Isler (1926–2009) are some of the most famous designers who used this method to establish structural shapes in their projects. Hanging models used by Gaudì are indicated in Fig. 2.

The second group is made of graphic methods, among which graphic statics is the most well known. This one is based on the dualism between the funicular polygon and the force polygon introduced by Varignon (see Fig. 3) and has been widely applied during 19th century by Karl Culmann (1821–1881), Luigi Cremona (1830–1903), James C. Maxwell (1831–1879), William Rankine (1820–1872) and Rafael Guastavino (1842–1908) [6,14,15].

The last and most recent group is made of numerical methods, developed from 1960, which can be subdivided between stiffness matrix methods, geometric stiffness methods and dynamic equilibrium methods depending on the computational approach adopted [16].

Nowadays computer aided models and digital morphogenesis models have also been developed, making more approachable the application of form finding methods to search the "right" structural shape. An example of a computer-aided model is given in Fig. 4 and is similar to the one used in the numerical studies explained in Section 5.

1.3. Funicularity evaluation

When no-bending occurs, the shape is determined by forces and vice versa. This is valid for 1D shapes, but it can be interpreted differently for shells. In theory, as confirmed by different authors, a shell properly supported can carry any load by membrane action only. Belluzzi [17] declares that "the behavior of a membrane differs from that of a cable. [....]The membrane is always in equilibrium for every external force and irrespective of its initial shape, and this equilibrium is satisfied solely by means of the internal membrane forces". In Pizzetti et al. [18], it is stated that "from a theoretical point of view, one could expect to oppose any curved thin surface to any load, confident that this surface will organize to perform statically at its best, that is a bending-free behavior." However bending-free behavior is only valid under two assumptions: the boundary conditions are congruent with the shape considered and the load applied, and the ratio between membrane stiffness and flexural stiffness tends to infinity; therefore a pure membrane model is applicable. In this circumstance the structure is locally isostatic and the equilibrium equations can be solved directly.

In practice, however, matters may not be so simple. Calladine [19] confirms that for a membrane "the nature of the solution may depend on the shape of the shell surface and the nature of the boundary conditions." Summarizing for a given shape and load, a funicular behavior can be found assigning the right boundary conditions to the shell analyzed; otherwise, for a given load and boundary conditions, the right shape needs to be found in order to obtain a no-bending behavior. Hence defining the "right" shape becomes crucial and different form finding techniques can be used for this purpose. Furthermore, when designing an actual surface, an elastic shell problem has to be solved: the flexural stiffness starts playing a role and the problem becomes hyperstatic. Bending moments, even if minimal, will arise. This also happens for form found shell surfaces when the thickness increases and consequently the ratio between membrane stiffness and flexural stiffness decreases.

The mutual dependency between shape and applied load leads to the common criticism made on the effective application of form finding methodologies; they can be suited to preliminary design, but the research of the shape cannot be generalized when the relevant load cases are multiple, since no-bending behavior cannot be guaranteed for all of them [4]. When this is the case and bending moments cannot be