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Elastic buckling of steel arches with discrete lateral braces

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ABSTRACT

Lateral **brace** is an effective way to increase the out-of-plane stability of steel arches, and the certain bracing stiffness achieving full restraining effect is concerned. Unlike braced columns and beams, the buckling of steel arches with discrete lateral braces and the effect of bracing stiffness are not well explored. This paper deals with the bracing stiffness effect of equally-spaced lateral translational braces and rotational braces in steel circular arches pertaining to elastic out-of-plane flexural-torsional buckling. Firstly, for arches with discrete lateral translational braces in uniform compression, an analytical solution for the threshold bracing stiffness is derived by the Rayleigh–Ritz method, ensuring full restraint of the displacement at the bracing points. Then for arches with discrete rotational braces in uniform compression, the flexural-torsional buckling modes are explored using the finite element analysis (FEA), then improved predictions to the buckling load related to the bracing stiffness are obtained theoretically by assuming proper buckling shapes. Compared with the FEA results, it is found that the analytical solutions of threshold stiffness proposed for arches in uniform compression are reasonably accurate and can be used in other loading cases for arches under combined compression and bending moment conservatively.

1. Introduction

Although the curved profile and the end thrust bring high-efficient in-plane load-carrying capacity, free-standing steel arches are prone to out-of-plane buckling that controls the strength design. The arrangement of lateral braces is an effectively way to increase the out-of-plane stability of arches. Threshold stiffness of lateral braces is an important factor defined as the minimum stiffness required to fully prevent the lateral deflections and twist rotations of cross-section at the bracing points. Beyond the threshold stiffness, the flexural-torsional buckling resistance of an arch will be increased little with the further increment of the bracing stiffness. In addition, the bracing strength is also required for an adequate brace arrangement, to withstand the reacting force at the bracing point [1,2].

A number of investigations have been reported in the open into the effect and design of lateral braces on columns and beams [3–11], as well as the flexural-torsional buckling of steel arches without lateral bracing. In contrast, studies on the out-of-plane behavior and bracing requirements of lateral-braced steel arches are rare. Bradford and Pi [12] explored the elastic buckling behavior of hinged arches in uniform bending and in uniform compression with a central torsional or lateral-translational restraint and proposed fitting equations for the full-bracing stiffness and buckling loads based in finite element analyses. Pi and Bradford [13] carried out an analytical study on the elastic out-of-plane buckling of continuously restrained arches using the energy approach and obtained closed form solutions of buckling loads. The number of half sine waves corresponding to the lowest buckling load (or moment) increases with the stiffness of the restraints, which is similar to the result for columns reported by Trahair [14]. Guo and Dou [15] studied I-sectional steel arches with multiple lateral translational braces based on the finite element analysis, and conservative solutions for predicting the threshold stiffness of lateral braces were suggested by a curve-fitting numerical procedure. Guo et al. [16] conducted an investigation into the flexural-torsional buckling of laterally braced circular arches, and the analytical expression of elastic threshold stiffness was derived using an energy approach. With respect to buckling in inelastic range, as far as the authors know, only Pi and Bradford [17] probed into the inelastic flexural-torsional buckling and strength of steel arches with a central torsional restraint, and it was found that due to the elasto-plastic property, the threshold restraint stiffness of an inelastic arches is smaller compared with an arch that buckles elastically.

Since the flexural-torsional buckling behaviour of an arch is quite

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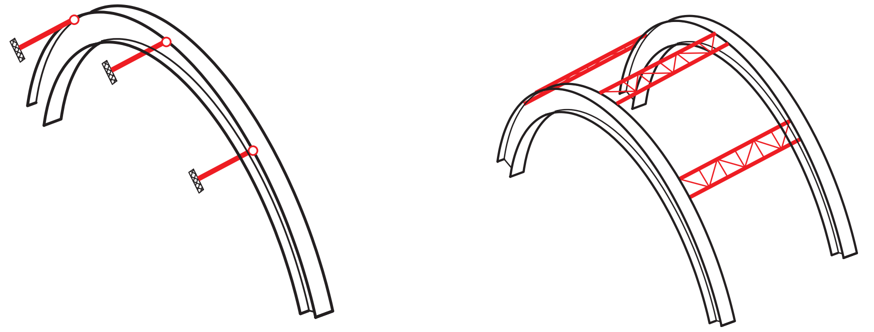
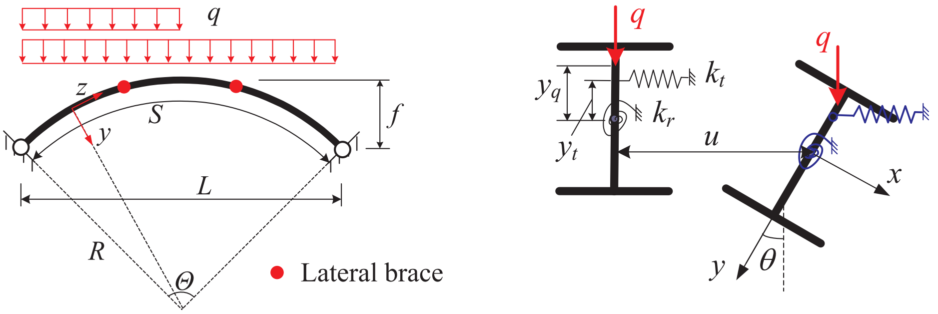
Nomenclature

A	cross-sectional area of sections
E	modulus of elasticity
EI_w	warping rigidity of cross-section
EI_y	flexural rigidity of cross-section with respect to out-of-plane bending
f	rise of an arch
G	shear modulus of elasticity
GJ	torsional rigidity of cross-section
$\overline{GJ}$	modified torsional rigidity of cross-section
I_y	out-of-plane second moment of area of the cross-section
i_y	$=\sqrt{I_x/A}$, gyration radius of the cross-section with respect to out-of-plane bending
K	sectional warping parameter
$k_{th,m}$	threshold bracing stiffness for m lateral translational braces in an arch
$k_{th,c}$	threshold bracing stiffness for a column with lateral translational braces
L	span of an arch
k_r	elastic bracing stiffness of discrete rotational braces
k_t	elastic bracing stiffness of discrete lateral translational braces
δ	amplification of the Fourier series for lateral deflection
ψ	amplification of the Fourier series for torsional deformation
N_{crt}	flexural–torsional buckling axial force of fixed arches with lateral translational braces
N_{crr}	flexural–torsional buckling axial force of fixed arches with rotational braces
q	distributed load on the arch
q_{cr}	distributed load corresponding to flexural–torsional buckling of an arch
N	axial compressive force of cross-section
h	overall height of cross-section
b	flange width of cross-section
P_{cr}	the first mode flexural buckling load of an axially-loaded column
m	the number of discrete braces
y_q	the coordinate of distributed loading position along the y axis on the cross-section
y_t	the distance of the bracing point away from the shear centre of the cross-section
R	radius of an arch
r_0	polar gyration radius of cross-section
S	developed length of cross-sectional centroid axis of an arch
t_f	flange thickness of cross-section
t_w	web thickness of cross-section
u	out-of-plane translational displacement of cross-sectional centroid of an arch
y_q	loading position of external force over cross-section along the y axis
Θ	subtended angle of an arch
θ	twist rotation of cross-section of an arch
λ_y	$=S/i_y$, out-of-plane slenderness of an arch
φ	coordinate of central angle of an arch
φ_i	the coordinate of arch central angle of the bracing point i

different from that of a beam-column, research findings relevant to the buckling of restrained beams and columns cannot be extended directly to arches. Due to the curved profile, the flexural-torsional buckling behaviour of a free-standing arch is more complex than that of a straight column or beam. By adding laterally braces, balanced equations for flexural-torsional buckling can hardly be established using the equilibrium approach, therefore the numerical approach based on the finite element analysis, as well as the Rayleigh–Ritz method based on the energy approach are the common ways adopted. Although equations were given for laterally braced arches of elastic



(a) Free-standing arch with lateral braces   (b) Twin arches with connecting members



(c) Analytical model for braced arches   (d) Buckling deformation of cross-section

Fig. 1. Circular arches with discrete lateral braces.