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Meshfree analysis of structures modeled as extensible slender rods

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ABSTRACT

The analysis of members that can be modeled as extensible elastic slender rods is investigated. A meshfree formulation using a Local Radial Point Interpolation Method (LRPIM) is developed that utilizes radial basis functions in curvilinear coordinates. This approach bypasses the need to utilize more conventional element meshes and significantly reduces the number of equations needed for the numerical solution. The slender rod formulation presented allows for tension variation, axial stretch, incremental loading and distributed load variation along the rod. It is well suited for nonlinear problems that involve large deflections and rigid rotations. The position and tangent vectors are expressed using Hermite-type approximations, and radial basis functions, while the interpolation of tension variation and distributed loads are described using polynomials. The solution procedure of weighted residuals Galerkin weak formulation combined with an incremental iterative numerical scheme is introduced to address the incremental loading and large deflection issues for static and quasi-static problems. The implementation of the analytical formulation and the numerical procedure are illustrated using three nonlinear problems. The first two examples provide insight into the validity, accuracy, and efficiency of the methodology. The third example presents the case of a moving boundary condition problem which models a cable entangled by fishing boat-trawling equipment.

1. Introduction

Slender rod/beam models of structures are widely employed in civil and offshore engineering. Examples include subsea power cables transmitting electricity, catenary cable in a suspension bridge, marine risers used to vertically recover hydrocarbons from subsea wells to floaters, and mooring line systems used for station-keeping of floating vessels. These applications typically adopt finite element modeling to predict the behavior of members. Meshfree methods do not require a fine grid/element mesh over the problem domain and consequently may have comparative advantage over finite element analysis for some nonlinear problems, e.g. large deflection problem induced by large rigid rotation and displacement. Since meshfree methods adopt a local approximation of field values using a group of neighboring nodes rather than nodes in a predefined finite element, there is no element distortion issue, and the domain representation in the proposed meshfree method can follow the instantaneous configuration of deformed slender rods. In recent years, Zhou [1–3] further developed and implemented catenary cable element using Finite Element Formulation in the dynamic response study and the cable-breakage event investigation of long-span cable-suspension bridges. By introducing joint node, the catenary cable element approach exhibits its merit on complex engineering structures embracing slender rod-like members with concentrated loads. Differently, the proposed method in this paper can solve problems with moving boundaries using the proposed numerical scheme.

In recent decades, innovative meshfree methods have been developed and implemented to solve 4th order partial differential equations (PDEs). Various capabilities of meshfree methods have been demonstrated in the solution of conventional structures that involve systems of beams, plates, and shells. Pioneers including Belytschko [4], Liu [5], Atluri [6], Liu [7] have lead the development of meshfree methods with the introduction of the element free Galerkin (EFG) method, the reproducing kernel particle method (RKPM), the meshless local Petrov-Galerkin method (MLPG), the point interpolation method (PIM), and the radial point interpolation method (RPIM), respectively. The differences between these methods include weak formulations and field interpolation techniques. These meshfree methods have been applied to thin and thick beams problems by a few scholars. Using the Reproducing Kernel Particle Method (RKPM) approach in the construction of shape functions for the field interpolation and constitutive law, Chen [8] obtained the solution for the large deformation of a thick beam involving the geometric and material nonlinearities. Donning [9] applied Galerkin weak formulation and RKPM interpolation scheme to a curved beam and a Mindlin plate. Subsequently, the MLPG was validated by a comparison to the analytical solution of Bernoulli-Euler beam theory [10]. By adopting the local weak formulation, the PIM was

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applied to a straight thin beam by Gu [11], which borrows the idea of the discretization of strong-form governing equation from MLPG. Cui [12] applied the gradient smoothing technique proposed by Liu [13] to obtain the solution of a thin beam, assuming that the rotation angle and the displacement as independent variables in the field value interpolation, while, the weak formulation was addressed using a gradient smoothing technique. Later Liu [14,15] studied the effects of shape parameters of RPIM and recommended the optimal parameters based upon comparative numerical studies of four common radial basis function families. The meshfree method adopts field nodes to represent the problem domain and it overcomes the difficulty of instantaneous re-meshing of deformed structures experiencing large deflections.

The evolution of numerical solutions for the large deflection of slender rod formulations dates back to the 1970s, where Nordgren [16] and Garrett [17] formulated the equation of motion of slender rods by vector analysis in a curvilinear coordinate and then obtained solutions using finite difference and finite elements respectively. Later Ma [18] extended this formulation to a flow line with internal pressure and under complex hydrodynamic loads and for offshore applications in a two-dimensional space. Subsequent to that Chen [19] introduced a new constraint condition allowing large elongation to tension dominant slender rod and further implemented the three-dimensional formulation in a program named Cable3D [20].

In this paper, a Local Radial Point Interpolation Method (LRPIM) is formulated to the extensible slender rod-like structure experiencing large deflection using Hermite-type radial basis function for field value approximations. Only a group of field nodes is utilized to describe the problem domain and no element is used in the formulation. Compared with a finite element formulation, the formulation presented reduced the N equations adopting as compared with the same number of field nodes as Finite Element Method. A local weak formulation is adopted in order to transform the partial differential equations into linear algebraic equations. Instead of using the deflection as independent variable of beam theories, the position vector using the arc length along the rod is employed as a primary variable. The tangent vector at specified points on the rod is also introduced as the additional variable in order to form a closed solution. Further, associated numerical issues are investigated such as the shape function, the shape parameters and the numerical convergence. The static analyses of a post-buckling column, and a catenary cable were performed to validate the methodology presented. A third example with moving boundary and varying cable length is introduced in order to demonstrate the capacity of the formulation to easily address cables with change in length and moving boundaries. Since the torsional stiffness is not addressed in the methodology presented, it is limited to engineering structures where torsion can be neglected. Practical offshore engineering examples include the global analysis of vertical risers for offshore oil and gas production and the interaction of subsea cables entangled with fishing gear.

2. Slender rod formulation

We begin by reviewing the slender rod formulation as initially presented by Nordgren [16] and Garrett [17], where uniform bending stiffness, no shear deformation and no rotational inertia are assumed. The instantaneous configuration of a cable is described by $\vec{r}(s,t)$ as shown in Fig. 1. The unit tangent vector, the unit normal vector, and the unit binormal vector are denoted as $\vec{t}$, $\vec{n}$ and $\vec{b}$, respectively. Some basis in the differential geometry of curves including the Serret Frenet formulae are utilized and the unit normals can be written as

$$\vec{t} = \vec{r}', \vec{n} = \vec{r}''/\kappa, \vec{b} = \vec{t} \times \vec{n} \tag{1}$$

where prime denotes the derivative with respect to s and κ is the curvature defined by an identity $\kappa^2 = \vec{r}''\cdot\vec{r}'' = -\vec{r}'\cdot\vec{r}'''$.

Fig. 2 illustrates a differential element on the slender rod, and according to the conservation of linear and angular momentum, the

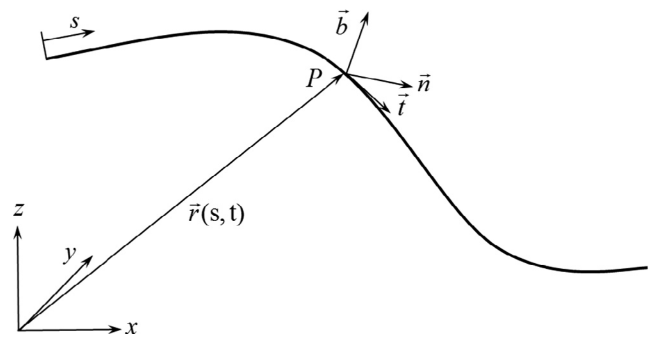

Fig. 1. Cartesian coordinate and curvilinear coordinate for a slender rod.

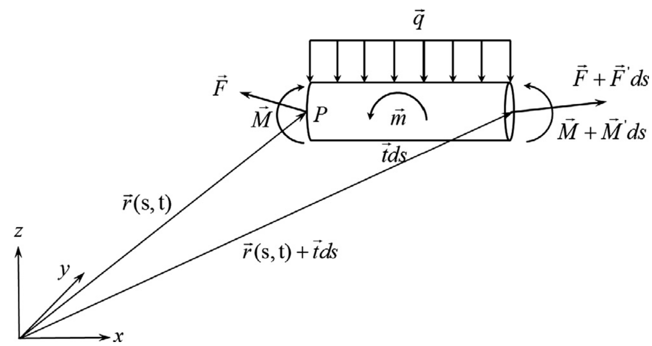

Fig. 2. A differential segment on a rod.

equations of motion can be expressed as

$$\vec{F}' + \vec{q} = \rho\ddot{\vec{r}} \tag{2}$$

$$\vec{M}' + \vec{t} \times \vec{F} + \vec{m} = 0 \tag{3}$$

where, ρ is the mass per unit length, $\vec{q}$ and $\vec{m}$ are the distributed force and moment along the rod per unit length respectively, and $\vec{F}$ and $\vec{M}$ are the internal force and moment of the cross section. The double dot notation indicates the second derivative with respect to time t.

Although small deformations were initially assumed, these equations allow for large deflection of the slender rod taking into account a small rotational angle caused by bending and resulted rigid body motion. Thus, the Bernoulli-Euler beam theory is still applicable to the constitutive law adopted here, and the bending moment and torque are proportional to curvature and twisting angle per unit length and can be expressed as

$$\vec{M} = EI\kappa\vec{b} + H\vec{t} = \vec{r}' \times (EI\vec{r}'') + H\vec{r}' \tag{4}$$

$$\vec{M}' = \vec{r}' \times (EI\vec{r}'')' + H'\vec{r}' + H\vec{r}'' \tag{5}$$

where, EI is the bending stiffness, $H = C\alpha$ is torque, C is the torsional rigidity, and α is the angle of twist per unit length.

Upon substituting Eq. (5) into Eq. (2), one obtains the following equation

$$\vec{r}' \times (EI\vec{r}'')' + H'\vec{r}' + H\vec{r}'' + \vec{r}' \times \vec{F} + \vec{m} = 0 \tag{6}$$

Evaluation of the cross product in Eq. (6) by $\vec{r}'$ yields the expression

$$\vec{F} = \lambda\vec{r}' - (EI\vec{r}'')' \tag{7}$$

Then substituting Eq. (2) into Eq. (7) leads to the equilibrium equation

$$-EI\vec{r}'''' + (\lambda\vec{r}')' + q(s) = \rho\ddot{\vec{r}} \tag{8}$$

where, $\lambda = T - EI\kappa^2$ is the Lagrange multiplier.

The governing equation expressed by Eq. (8) is derived based on the