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Modeling and nonlinear dynamic analysis of cable-supported bridge with inclined main cables

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ABSTRACT

This work studies the dynamic properties and non-linear responses of a sectional model of long-span cable-supported bridge deck under scenarios of resonance with vortex induced vibration or wind fluttering. The cable supported bridge with spatially inclined cables is modeled by a 6-degrees-of-freedom system. The equations of motion for an undamped system are formulated for the study of the effects of inclination angle of cable on the modal parameters. The inclination angle is found strongly affecting the torsional modal frequency of the deck. The equations of motion on the forced excitation of a damped system are also developed for the study of non-linear responses when in resonance. The response analysis is conducted with the Incremental Harmonic Balance method. The inclination angle does not have notable nonlinear effects on the primary resonance of deck heaving response with vertical excitation as the responses of the system are almost identical with those obtained through the linearized model. However, the primary resonance response due to periodic pitching moment indicates an increasing nonlinear effect with an increase in the cable inclination angle. The cables vibrate strongly in both the horizontal and vertical directions with multiple frequency components and non-zero stationary component. The super-harmonic resonance of deck heaving and rotation can only be observed when the damping ratio of system is extremely low.

1. Introduction

Oscillation of long-span cable-supported bridges, such as the vibration under aerodynamic forces [1–3] and moving automobiles [4–6], is an important safety issue for the structure. Many mathematical models have been developed to investigate different scenarios of vibration of this kind of structure system.

Some researchers adopted a continuum model, which was originally designed for the study of linear vertical vibrations of cable-supported bridges. The stiffened truss girder was modeled by an Euler–Bernoulli beam, and the main cables supported the bridge deck through inextensible and distributed vertical hangers. Bleich et al. [7] proposed the classic continuum model based on the linearized deflection theory. The linear theory of a suspended elastic cable transformed a taut string into an inextensible suspended cable with a small sag [8]. Luco and Turmo [9] re-examined the classical continuum modeling approach with an extension for the study of modal frequencies, mode shapes, and modal participation factors of an extensible suspension cable. Abdel-Rohman [10] studied the influence of higher order vibration modes on the dynamic response of a cable-supported bridge. It was shown that the influence of the higher order modes, when the suspension bridge is subjected to wind loading, is more significant than in the case when the bridge is subjected to a moving load. Besides the above studies on a linear system, the vibration based on non-linear models was also studied [11–13]. Ding [14,15] studied the periodic oscillations in a cable-supported bridge system under periodic external forces. Malik [16,17] formulated the nonlinear model of a cable-supported bridge structure from the principle of minimum potential energy to describing the behaviour of bridge deck with discussions on the solution stability criterion.

Other researchers developed sectional models which can capture the essential characteristics of the dynamic behavior of the structure. They comprise of rigid bodies and springs. The occurrence of high-amplitude oscillation can often be related to the modal properties of the structure, and many large vibrations are related to the resonance of the structure. These sectional models have been demonstrated to be capable to illustrate the interaction performance between oscillations in the torsional and vertical directions, though in an approximate and analytical form. Two Degrees-of-Freedom (DoFs) system has been adopted in many studies for the analysis of rotational response of the bridge deck

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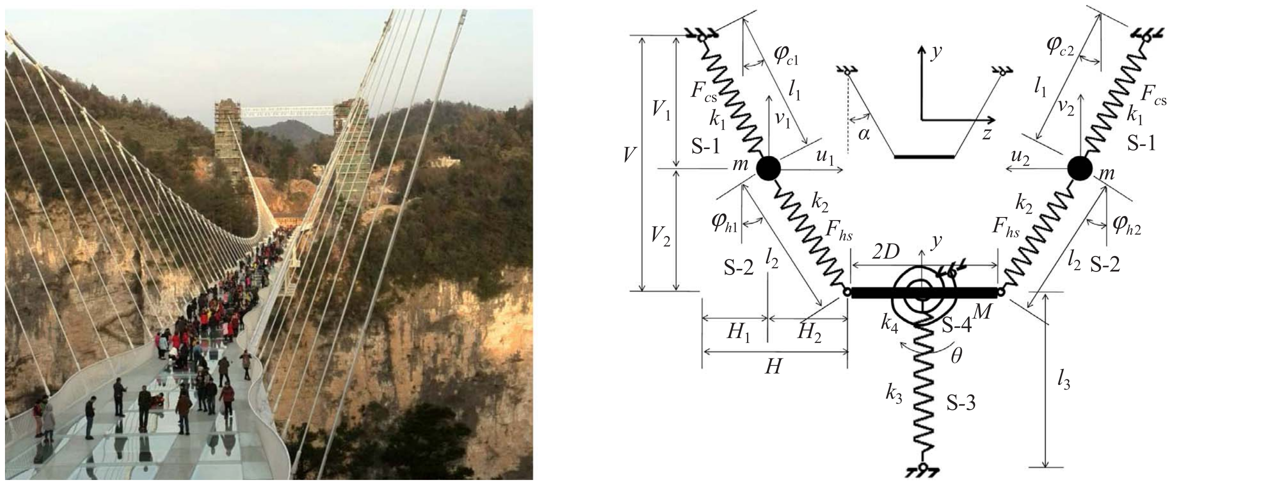

(a) Pedestrian cable-supported bridge (b) Sectional model

Fig. 1. Model of cable-supported bridge with spatially inclined cables. (a) Pedestrian cable-supported bridge; (b) Sectional model.

[18–21]. Plaut and Davis [22] extended the two-DoFs model into four-DoFs model by introducing two more DoFs which represent the vertical motion of the main cables on two sides. De Freitas et al. [23,24] used the Lazer-McKenna model to study the nonlinear properties of the system under periodic external forces. More recently, a multi-body system was proposed to model the bridge section of long span cable-supported bridges for both linear and nonlinear dynamic studies [25–27]. This four-DoFs system accounts for both the vertical-torsional motion of the bridge deck and the transversal motion of a pair of hangers or stay cables. The interaction between the motions of the bridge deck and stay cables was systematically investigated.

Although various models have been proposed to investigate the linear and nonlinear dynamics of cable-supported bridge, yet all these models were based on the traditional bridge configurations with the main cables and hangers in a vertical plane supporting the bridge deck below. It should be noted that some new bridges have been constructed in the last few decades where the main cables supporting the bridge deck have a spatial geometric layout with inclined angle to the horizontal direction. One of such bridge is shown in Fig. 1(a). The cables support the deck in both the vertical and lateral directions, whereby the stiffness coupling of the cables and deck would significantly affect the modal properties of the structure.

There has not been any study on the resonance of such cable-supported bridge deck when under vortex induced vibration or flutter. The effect of geometric nonlinearity in the new type of cable supporting system may be significant compared to that with vertical cable support. Thus this new type of bridge structure requires detailed study on its dynamic properties. A six-DoFs model is developed in this paper for this type of cable-supported bridge, and its dynamic properties with different inclination angle of the cables are investigated. The non-linear vibration in the responses is studied when the structure is in resonance. The equation of motion of a free undamped system and a damped system under external excitation are formulated. They are solved with the Incremental Harmonic Balance method for the study of the non-linear dynamic responses of the cables and deck in two directions.

2. Model of the system

The proposed model is a 6DoFs system comprising of three masses linked up by six springs as shown in Fig. 1(b). The masses are from the sectional bridge deck and the two cables. The six springs are classified into four types: S-1 denotes the in-plane stiffness of the main cable; S-2 denotes that of the hanger; S-3 and S-4 denote the vertical and torsional stiffnesses provided by the bridge deck. This model can be used to study all the motions of the sectional bridge deck and cables in the y-z plane, and the contributions from the cables, hangers and deck to the system can be accounted for. All variables in the figure are defined as follows:

V_i (i = 1,2)	Projection of S-i (i = 1,2) in vertical direction
H_i (i = 1,2)	Projection of S-i (i = 1,2) in horizontal direction
D	Half of the breadth of bridge deck
α	Representative inclination angle of cable and hanger, $\alpha = \arctan\frac{H_1+H_2}{V_1+V_2}$
l_i (i = 1,2,3)	Length of spring S-i (i = 1,2,3)
m	Mass of cable
M	Mass of bridge deck
J	Moment of Inertia of bridge deck
k_i (i = 1,2,3,4)	Stiffness of spring S-i (i = 1,2,3,4)
u_i (i = 1,2)	Horizontal DoF of m
v_i (i = 1,2)	Vertical DoF of m
y	Vertical DoF of M
θ	Rotational DoF of M
F_{cs}	Internal force of S-1 under static condition
F_{hs}	Internal force of S-2 under static condition
φ_{ci}	(i = 1,2) Angle between S-1 on two sides and the vertical plane
φ_{hi}	(i = 1,2) Angle between S-2 on two sides and the vertical plane

The equations of motion governing the free undamped vibration of the system can be written as follows:

$$M\ddot{y} + k_3y-(F_{hs} + F_{hd1})\cos(\varphi_{h1})-(F_{hs} + F_{hd2})\cos(\varphi_{h2}) + Mg = 0 \tag{1a}$$

$$J\ddot{\theta} + k_4\theta-(F_{hs} + F_{hd1})\cos(\varphi_{h1})D\cos\theta + (F_{hs} + F_{hd2})\cos(\varphi_{h2})D\cos\theta + (F_{hs} + F_{hd1})\sin(\varphi_{h1})D\sin\theta + (F_{hs} + F_{hd2})\cos(\varphi_{h2})D\sin\theta = 0 \tag{1b}$$

$$m\ddot{u}_1 + (F_{cs} + F_{cd1})\sin(\varphi_{c1})-(F_{hs} + F_{hd1})\sin(\varphi_{h1}) = 0 \tag{1c}$$

$$m\ddot{v}_1 + mg-(F_{cs} + F_{cd1})\cos(\varphi_{c1}) + (F_{hs} + F_{hd1})\cos(\varphi_{h1}) = 0 \tag{1d}$$

$$m\ddot{u}_2 + (F_{cs} + F_{cd2})\sin(\varphi_{c2})-(F_{hs} + F_{ld2})\sin(\varphi_{h2}) = 0 \tag{1e}$$

$$m\ddot{v}_2 + mg-(F_{cs} + F_{cd2})\cos(\varphi_{c1}) + (F_{hs} + F_{hd2})\cos(\varphi_{h1}) = 0 \tag{1f}$$

where F_{cdi} and F_{hdi} (i = 1,2) are the dynamic spring forces in springs S-1 and S-2 on two sides. F_{cs} and F_{hs} can be obtained from the static balance equation of the system in both horizontal and vertical directions under the gravity load. It can be noted that when φ_{ci} and φ_{hi} (i = 1,2) are equal 0°, the model will represent a bridge with vertical cables and