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Mode shape scaling and implications in modal identification with known input

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ABSTRACT

This study proposes a mode shape scaling and parameterization scheme for modal identification with known input. Through the derivation of the equations for known input modal identification using the proposed mode shape scaling and parameterization scheme, the study provides insight into the relationship between the identified modal parameters and information required in the forced vibration test. In typical applications of modal identifications, when there is sufficient amount of data, the formulation using the proposed mode shape scaling and parameterization scheme shows that it allows modal parameters to be determined efficiently in a globally identifiable manner. An illustrative example using synthetic data is provided in this study. The findings show that an appropriate mode shape scaling and normalization scheme could reduce the information required in the modal identification procedure for some modal parameters, i.e. natural frequencies, damping ratios and mode shapes. This significantly simplifies the procedure of the forced vibration test, and hence, it can be carried out in a more robust manner.

1. Introduction

Modal identification is a technique that allows extraction of the modal parameters, such as natural frequencies, damping ratios, and mode shapes, of a structure from measured vibration data [1]. The identified modal parameters can then be used for structural model updating [2] and damage detection [3]. In the last decade, vibration tests have been carried out on different types of structures, e.g. bridge [4], tower [5] and building [6].

Forced vibration test makes use of a special device, such as shaker or impact hammer, to produce vibration response of structures for identifying modal properties. Memari et al. [7] carried out a forced vibration study on a six story steel frame building during the construction stage. The forced vibration was carried out when steel frames, floor slabs and some of the walls were completed. An unbalanced mass exciter was installed at the roof of the building to induce the excitation. Natural frequencies, damping ratios and mode shapes were identified from the measured acceleration data. Halling et al. [8] conducted a forced vibration test on a concrete deck steel girder bridge. An eccentric mass shaking machine was used to generate the required excitation on the bridge. The study identified the natural frequencies and mode shapes. These identified modal parameters were also used to update a finite element model of the bridge. Burgueno et al. [9] carried out a forced vibration test on a fiber reinforced polymer (FRP) composite bridge. They employed a long stroke electro-dynamic force generator to excite the bridge and the measured acceleration data was used to identify the natural frequencies and mode shapes.

Although more demanding in terms of budget and logistics, forced vibration test has several advantages over free [10] or ambient vibration tests. Essentially, the signal-to-noise ratio of data can be significantly improved and the information of input excitation can significantly reduce the identification uncertainty of modal parameters [11]. In typical applications, the location and direction of the artificial excitation is assumed to be known, although in some cases it is difficult to control them in field testing conditions [12].

The objective of this study is to demonstrate that an appropriate mode shape scaling scheme can reduce the information required in the modal identification procedure for some modal parameters, such as natural frequencies, damping ratios and mode shapes, allowing forced vibration tests to be performed in a more robust manner. A mode shape scaling and parameterization scheme is first proposed, which allows modal parameters to be determined efficiently in a globally identifiable manner. Based on this scheme, implications on the required information in the modal identification are discussed. A Bayesian context is assumed as it allows uncertainties to be fundamentally quantified, but the implications on identifiability are general and applicable to other

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non-Bayesian or deterministic approaches.

Section 2 first summarizes the formulation of the known input modal identification. Section 3 proposes the mode shape scaling and parameterization scheme and its formulation of the known input modal identification. Section 4 discusses the relationship between modal identification and the information of the exciter configuration. Section 5 presents the formulation of the Bayesian approach under the proposed mode shape scaling and parameterization scheme. Insights and practical aspects are discussed in Section 6. Section 7 presents an illustrative example. Finally, conclusions are provided in Section 8.

2. Modal identification with known single input

Consider a multi-degree-of-freedom (MDOF) structure satisfying the dynamic equation:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are respectively the conventional mass, damping, stiffness matrices; and $\mathbf{F}(t)$ is the force vector. With mass normalization, the i-th modal force is given by

$$p_i(t) = \frac{\boldsymbol{\varphi}(i)^T\mathbf{F}(t)}{\boldsymbol{\varphi}(i)^T\mathbf{M}\boldsymbol{\varphi}(i)} \tag{2}$$

and $\boldsymbol{\varphi}(i) \in R^n$ is the (partial) mode shape vector of the ith mode confined to the n measured dofs. Without loss of generality, suppose the acceleration response of the structure is measured at n degrees of freedom (dofs). Assuming m contributing modes, the measured data in the frequency domain can be modeled as

$$\mathscr{F}_k = \sum_{i=1}^{m} \boldsymbol{\varphi}(i) h_{ik} P_{ik} + \boldsymbol{\varepsilon}_k \tag{3}$$

where $\mathscr{F}_k$ is the fast Fourier transform (FFT) of measured data at frequency $\mathrm{f}_k = k/N\Delta t$ (Hz); N is the number of samples per data channel; Δt is the time step; P_{ik} is the FFT of the modal force at k-th frequency; $\boldsymbol{\varepsilon}_k$ is the prediction error (e.g., measurement noise). For a given mode i, h_{ik} is the transfer function between modal excitation and modal acceleration:

$$h_{ik} = -[(\beta_{ik}^2 - 1) + \mathbf{i}(2\zeta_i\beta_{ik})]^{-1} \tag{4}$$

where $\beta_{ik} = f_i/\mathrm{f}_k$ is a frequency ratio; $\mathbf{i}^2 = -1$. f_i (Hz) and ζ_i are respectively the natural frequency and damping ratio.

During testing measurement, suppose the structure is subjected to a single dominant source of artificial excitation that is also measured. Depending on the direction of the applied excitation on the structure, the force can be distributed to more than one dof. For convenience in analysis, assume without loss of generality that the force on the jth measured dof is given by $ma_j s(t)$, where m (kg) is a nominal mass value (e.g., moving mass of a shaker), a_j is a dimensionless factor accounting for the contribution of force to the dof, which has value between 0 and 1, and it is zero on other unmeasured dofs. $s(t)$ (m/s^2) is a time-varying function of excitation (e.g., acceleration of shaker mass). In this context, $\boldsymbol{\varphi}(i)^T\mathbf{F}(t) = ms(t)\boldsymbol{\varphi}(i)^T\mathbf{a}$, where $\mathbf{a} = [a_1,...,a_n]^T$. The modal force and its FFT are given by

$$p_i(t) = r_i[\boldsymbol{\varphi}(i)^T\mathbf{a}]s(t) \tag{5}$$

$$P_{ik} = r_i[\boldsymbol{\varphi}(i)^T\mathbf{a}]S_k \tag{6}$$

where S_k is the FFT of $s(t)$ and

$$r_i = \frac{m}{\boldsymbol{\varphi}(i)^T\mathbf{M}\boldsymbol{\varphi}(i)} \tag{7}$$

is the ratio of nominal mass to the modal mass. Substituting Eqs. (4) and (6) into Eq. (3), we have

$$\mathscr{F}_k = \sum_{i=1}^{m} S_k h_{ik} r_i [\boldsymbol{\varphi}(i)^T\mathbf{a}]\boldsymbol{\varphi}(i) + \boldsymbol{\varepsilon}_k \tag{8}$$

3. Mode shape scaling and parameterization scheme

Eq. (8) is the basic equation that relates the data $\{\mathscr{F}_k\}$ and $\{S_k\}$ to modal parameters. The modal parameters include, for each mode, f_i (natural frequency), ζ_i (damping ratio), r_i (modal mass ratio) and $\boldsymbol{\varphi}(i)$ (mode shape); and parameters defining the statistical properties of the prediction error. The mode shape is subjected to a scaling constraint.

Using Eq. (8) directly to identify the modal parameters does not lead to an effective scheme, primarily because of its quadratic dependence on mode shape $\boldsymbol{\varphi}(i)$, which is also subjected to scaling constraint. For example, Eq. (8) will lead to a fourth-order dependence on mode shape in the objective function of a least square approach. Here, a mode shape scaling and parameterization scheme is proposed that allows the parameters to be determined efficiently in a globally identifiable manner and reduce the information required in the modal identification procedure. Beyond significance of computational nature, an interesting implication of the scheme is that the identification results are found to be invariant to the vector $\mathbf{a}$, which reflects the location and orientation of the artificial excitation. These practical implications shall be discussed in Sections 4–6.

Conventionally, mode shapes may be scaled to be 1 at a particular dof or to have unit norm [13–16]. Neither of these can eliminate the quadratic dependence in Eq. (8) on mode shape. Upon investigation of the mathematical structure of the problem, it is found that the following scaling constraint allows the problem to be resolved while allowing for flexible implementation without prior information

$$\boldsymbol{\varphi}(i)^T\mathbf{a} = 1 \quad \text{for } i = 1,...,m \tag{9}$$

so that

$$\mathscr{F}_k = \sum_{i=1}^{m} S_k h_{ik} r_i \boldsymbol{\varphi}(i) + \boldsymbol{\varepsilon}_k \tag{10}$$

becomes a linear function of $\boldsymbol{\varphi}(i)$. Note that r_i and $\boldsymbol{\varphi}(i)$ are subjected to the constraint $\boldsymbol{\varphi}(i)^T\mathbf{a} = 1$. The formulation can be further simplified by combining them into an unconstrained vector

$$\boldsymbol{\varphi}_r(i) = r_i\boldsymbol{\varphi}(i) \tag{11}$$

so that

$$\mathscr{F}_k = \sum_{i=1}^{m} S_k h_{ik} \boldsymbol{\varphi}_r(i) + \boldsymbol{\varepsilon}_k \tag{12}$$

The parameters to be identified now comprise, for each mode i, f_i, ζ_i, $\boldsymbol{\varphi}_r(i)$ with no constraint; and parameters specifying the statistical properties of $\boldsymbol{\varepsilon}_k$. Once these parameters are identified, the modal mass ratio can be recovered by using Eqs. (9) and (11)

$$r_i = \boldsymbol{\varphi}_r(i)^T\mathbf{a} \tag{13}$$

4. Invariance to exciter configuration

In addition to providing an effective formulation for modal identification, the mode shape scaling and parameterization scheme in Section 3 also leads to an interesting implication on how identification results depend on exciter configuration. Specifically, for given data ($\{\mathscr{F}_k\}$ and $\{S_k\}$), the information of $\mathbf{a}$, which is related to exciter location and orientation, is not needed to identify f_i, ζ_i and $\boldsymbol{\varphi}_r(i)$ as shown in Eq. (12) and the same for other parameters related to the statistical modeling of prediction error. However, different values of $\mathbf{a}$ do affect the identification results because it affects the excitation magnitude. Although the scaling constraint on $\boldsymbol{\varphi}(i)$ in Eq. (9) depends on $\mathbf{a}$, $\boldsymbol{\varphi}(i)$ is in fact invariant because it has the same 'shape' as $\boldsymbol{\varphi}_r(i)$. To see this, $\boldsymbol{\varphi}(i) = r_i^{-1}\boldsymbol{\varphi}_r(i)$ makes the same hyper-angle with any vector $\mathbf{u}$ as $\boldsymbol{\varphi}_r(i)$:

$$\frac{\boldsymbol{\varphi}(i)^T\mathbf{u}}{||\boldsymbol{\varphi}(i)||\ ||\mathbf{u}||} = \frac{r_i^{-1}\boldsymbol{\varphi}_r(i)^T\mathbf{u}}{r_i^{-1}||\boldsymbol{\varphi}_r(i)||\ ||\mathbf{u}||} = \frac{\boldsymbol{\varphi}_r(i)^T\mathbf{u}}{||\boldsymbol{\varphi}_r(i)||\ ||\mathbf{u}||} \tag{14}$$