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A composite beam theory for modeling nonlinear shear behavior

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ABSTRACT

Accurate predictions of physically nonlinear elastic behaviors of a material point in the structure are essential to the further analyses which are beyond the linear elasticity regime, for example, the progressive damage and the failure. In light of substantial experimental evidence of nonlinear shear stress-strain responses in composites, it is necessary to consider them in the structure-level simulations rigorously. A variational asymptotic beam model is developed for this purpose. The three-dimensional continuum is rigorously reduced to a two-dimensional cross-sectional analysis and a one-dimensional Euler-Bernoulli beam analysis. The original three-dimensional continuum features material nonlinearities in longitudinal shear. The unknown cross-sectional warping is solved by finite element method using the principle of virtual work. Nonlinear beam constitutive relation and three-dimensional stress and strain fields are obtained.

1. Introduction

Fiber-reinforced plastic composites (FRP) exhibit physically nonlinear behaviors in both elastic and inelastic regions. The primary cause of the elastic nonlinearity has been discovered to be the lamina shear stresses once they are relatively large compared to the longitudinal tensile stresses. In this situation, the resin matrix dominates in the mechanical performance of the composites. Consequently, because the shear stress-strain responses of polymer resins are nonlinear over the entire strain range and at very low strain levels, the in-plane shear responses of FRP plies are nonlinear over the entire range examined [1].

Several mathematical models have been published to describe the nonlinear stress-strain responses. A widely used example of these models are developed by Hahn and Tsai [2] by employing a plane-stress complementary energy function which contains a biquadratic term for in-plane shear stress. Stress field predicted by such a constitutive law is used to formulate the failure criterions which are successful in predicting the failure due to stress concentrations [3,4]. Another widely used model is the Ramberg-Osgood equation [5] which is also popular in metal fatigue studies. A more flexible description methodology is to utilize mathematical curve fitting functions [6–8]. A comprehensive review of the nonlinear constitutive models for shear nonlinearity can be found in [9].

The focus of this paper is not to just provide another method to describe the nonlinear shear stress-strain law but to bridge the theoretical gap between the physically nonlinear laws and the mechanics of slender solid made of the materials which are governed by these laws. We have two main motivations for this study. Firstly, the knowledge of the nonlinear shear stress-strain response of the composites can be obtained from the measurements of loaded slender coupons. For example, the ASTM D3518/D3518M Standard Test Method [10] for "in-plane shear response of polymer matrix composite materials by the tensile test of ± 45° laminate" is based on the measured uniaxial force-strain response of a symmetrically ± 45°-laminated coupon. A rigorous beam model can serve as a virtual coupon to relate the uniaxial force-strain response precisely with the three-dimensional (3D) stress and strain fields by the cross-sectional analysis. Consequently, the beam model can be used along with the data matching tools to calibrate the material constants built into the material descriptions. Secondly, the nonlinear in-plane shear responses have impacts on the one-dimensional (1D) constitutive responses of beams. Predictions of static failure loads and natural frequencies of composite beams are affected by the predefined 3D nonlinear stress-strain laws.

A substantial amount of work has been devoted to model composite beams. The conventional beam theories adopt the ad hoc assumptions, for example, the cross section remains rigid in its own plane and possesses uniaxial stress state, have limited their generality and accuracy in predicting the behavior of composite beams. An advanced theory should be free from the limitation of unnecessary kinematic assumptions and minimize the information loss from the original 3D model.

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The variable kinematic model, known as the Carrera Unified Formulation (CUF) [11], permits one to develop a structural model with a variable number of displacement unknowns in a hierarchical manner. Another systematic approach for modeling composite beams has been developed by Hodges and his co-workers during the last three decades [12–17]. This approach uses the Variational Asymptotic Method (VAM) [18] to rigorously split the original 3D geometrically nonlinear and materially linear problem of the slender structure into a 1D global beam analysis and a two-dimensional (2D) cross-sectional analysis. The 2D cross-sectional analysis is called Variational Asymptotic Beam Sectional Analysis (VABS). The advantageous feature of this approach is that the resulting beam models are still in the form of simple engineering models such as the Euler-Bernoulli beam model or the Timoshenko beam model without the ad hoc assumptions such as that the plane cross section remains plane associated with these models. VABS provides the constitutive relations needed for the global 1D beam analysis and computes the pointwise fields (such as stress and strain) within the original 3D structure based on the global beam behavior. The nonlinear elasticity is studied in VABS framework firstly by assuming nonlinear strain definition (Green strain in St-Venant/Kirchoff model) to examine the trapeze effect for strip-like beams [19]. Jiang, Yu, and Hodges extended the VABS theory to deal with various types of hyperelastic material both analytically [20] and numerically [21]. The motivations to use VABS instead of 3D FEA is the computational efficiency, numerical stability, coupon constitutive relation representative, and flexibility for complex composite mold profile.

In the present work, VABS is extended to model the physically nonlinear beams. In light of releasing the small warping assumption to the finite warping, the nonlinear product terms of the warping and curvatures are retained in the strain formulation. The theoretical foundation of VABS is updated from minimizing the strain energy to the principle of virtual work. Newton-Raphson method is utilized to solve for the converged warping solution iteratively.

The Hahn-Tsai [2] nonlinear in-plane shear model is used for validation purpose by comparing the VABS results with those from 3D FEA. Both static and dynamic examples are given. The ± 45°-laminated coupon tensile tests are simulated. 3D local fields such as the free-edge stresses are precisely captured by the present model. Nominal stress-strain curves predicted for various composite beams with different cross-sections are compared to show the impact of the cross-sectional designs of the coupons on their performances in calibrating the material constants.

2. Variational asymptotic beam sectional analysis (VABS)

2.1. Theoretical formulation

In Fig. 1, $\boldsymbol{e}_i$ for $i = 1, 2, 3$ are fixed dextral, mutually perpendicular unit vectors in the absolute reference frame, and $\boldsymbol{r}_0$ and $\boldsymbol{R}_0$ denote the position vector of the material point on the reference line of the undeformed and deformed configurations, respectively. $\boldsymbol{b}_i$ and $\boldsymbol{B}_i$ are the orthogonal triads attached to the cross-section in the undeformed and deformed configurations, respectively. Here and through all the paper, expect where explicitly indicated, Greek index α assumes values 2 and 3, whereas Latin indices (i, j, k, l, m, n, p, and q) assume values 1, 2, and 3. Repeated indices are summed over their range except where explicitly indicated.

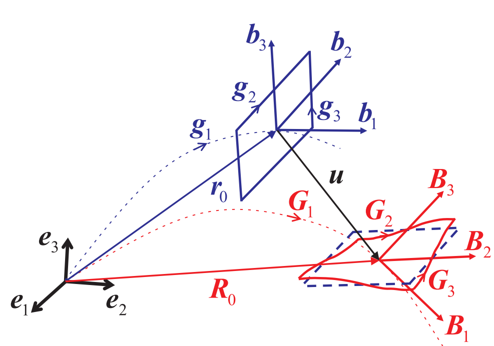

Fig. 1. Schematic of undeformed and deformed beam.

The material position vectors in the undeformed and deformed beam body can be expressed as

$$\boldsymbol{r} = \boldsymbol{r}_0 + x_\alpha \boldsymbol{b}_\alpha \tag{1}$$

$$\boldsymbol{R} = \boldsymbol{R}_0 + x_\alpha \boldsymbol{B}_\alpha + w_i(x_1,x_2,x_3)\boldsymbol{B}_i \tag{2}$$

with w_i representing the 3D unknown warping functions to describe the difference between the position of deformed body and those can be described by deformation of the reference curve x_1 in terms of $\boldsymbol{R}_0 + x_\alpha \boldsymbol{B}_\alpha$. $\boldsymbol{R}_0$ can also be expressed as

$$\boldsymbol{R}_0 = \boldsymbol{r}_0 + \boldsymbol{u} \tag{3}$$

where $\boldsymbol{u}$ denotes the beam displacement. Note $\boldsymbol{u}$ is not the displacement of some material point in the original structure. Rather it is the displacement field of the beam model (points on the beam reference line) we are constructing. In Eq. (2), we actually express $\boldsymbol{R}$ in terms of $\boldsymbol{R}_0$, $\boldsymbol{B}_i$, and w_i, which is six times redundant. Six constraints are needed to ensure a unique mapping. We can choose $\boldsymbol{B}_1$ to be tangent to the deformed reference line which introduces two constraints since we are building a model of Euler-Bernoulli type. As discussed in [16], we can also introduce the following four constraints for the warping functions:

$$\langle w_i \rangle = 0, \quad \langle w_{2,3} - w_{3,2} \rangle = 0 \tag{4}$$

From here and throughout the paper we assume a prismatic beam with uniform cross-sectional geometry. To derive a theory of the classical (Euler-Bernoulli) type, we define the following generalized 1D strains:

$$\boldsymbol{R}_0' = (1 + \gamma)\boldsymbol{B}_1 \tag{5}$$

$$\boldsymbol{B}_i' = \kappa_j \boldsymbol{B}_j \times \boldsymbol{B}_i \tag{6}$$

in which the upper prime denotes derivative to x_1, γ the axial strain, κ_1 the twist and κ_α the curvature of the deformed beam reference line. It is noted that these definitions of beam strains have nothing related with the well-known Euler-Bernoulli assumptions. Instead, we are constructing a model which is capable of capture extension (γ), torsion (κ_1), and bending in two directions (κ_α) with the possibility to capture all the 3D displacements, strains, and stresses due to these four fundamental deformation modes allowed in the Euler-Bernoulli beam model without apriori assuming that some components of the 3D fields vanish as most other theories do.

In Fig. 1, $\boldsymbol{g}_i$ denote the covariant base vectors of the undeformed body. And let the controvariant base vectors of the undeformed body denoted by $\boldsymbol{g}^i$. Then we have

$$\boldsymbol{g}^i = \boldsymbol{g}_i = \boldsymbol{b}_i \tag{7}$$

for prismatic beams. The covariant base vectors of the deformed configuration can be evaluated as

$$\boldsymbol{G}_k = \frac{\partial \boldsymbol{R}}{\partial x_k} \tag{8}$$

Together with Eq. (6), we have

$$\begin{aligned}\boldsymbol{G}_1 = {} & [1 + \gamma + w_1' - (x_2 + w_2)\kappa_3 + (x_3 + w_3)\kappa_2]\boldsymbol{B}_1 \\ & + [w_2' - (x_3 + w_3)\kappa_1 + w_1\kappa_3]\boldsymbol{B}_2 \\ & + [w_3' + (x_2 + w_2)\kappa_1 - w_1\kappa_2]\boldsymbol{B}_3\end{aligned} \tag{9}$$

$$\boldsymbol{G}_2 = w_{1,2}\boldsymbol{B}_1 + (1 + w_{2,2})\boldsymbol{B}_2 + w_{3,2}\boldsymbol{B}_3 \tag{10}$$

$$\boldsymbol{G}_3 = w_{1,3}\boldsymbol{B}_1 + w_{2,3}\boldsymbol{B}_2 + (1 + w_{3,3})\boldsymbol{B}_3 \tag{11}$$

Then the deformation gradient tensor can be formulated as the dyadic