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A closed-form solution of the interfacial stresses and strains in steel beams strengthened with externally bonded plates using ductile adhesives

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ABSTRACT

Ductile adhesives are known to be beneficial in enhancing the capacity and ductility of bonded joints. However, there is a lack of closed-form analytical solutions for plated metallic beams that account for adhesive plasticity. This paper presents a first order, elastic-plastic bond analysis for beams strengthened with externally bonded fiber reinforced polymer (FRP) plates using ductile adhesives, based on a shear-lag formulation. This model is able to analyze arbitrary mechanical and thermal loading conditions and closed-form solutions under shearing and peeling are given. Following a review of the existing stress-based analytical solutions, a comparison between the existing and proposed analyses is also presented. The shear solution was validated by comparing with the experimental results of carbon fiber reinforced polymer (CFRP) strengthened steel beams under moderately elevated temperatures. A comprehensive parametric study has been conducted to illustrate the effects of different design parameters on the bond behavior. It is found that the adhesive shear toughness is the most critical parameter in determining the debonding failure load while the adhesive Young's modulus does not significantly affect the bond stresses in the elastic-plastic regime. Further, the magnitude of the peak peeling stresses is self-limiting after shear yielding, and the use of a thinner plate with higher Young's modulus is beneficial in further reducing the peeling stress.

1. Introduction

Flexural strengthening of beams with externally bonded plates has been widely researched [1–5]. The design of the strengthened beams should consider other potential failure modes, such as lateral torsional buckling [6–8], or shear failure to preclude these failure modes at the increased load capacity of the flexurally strengthened beams. Two failure modes that are important to consider for fiber reinforced polymer (FRP) strengthened steel beams include FRP debonding and rupture. The FRP rupture failure load can be easily analyzed through sectional analysis assuming full interaction between the FRP plate and the beam. For FRP strengthened steel beams the debonding failure occurs either at the bonded interfaces (adhesive failure) and/or within the adhesive layer (cohesive). Therefore, the interfacial stress analysis within the adhesive layer is important to characterize the stress transfer and potential for debonding in plated steel beams.

In order to predict the debonding failure, analytical models have been proposed to solve for the stresses within the adhesive layer. The early stages of developing the analytical solutions for plated beams primarily focused on steel plate strengthened concrete beams, and linear elastic material properties of the adhesive were assumed.

Vilnay [9] provided solutions for the shear and peel stresses of concrete beams strengthened with epoxy bonded steel plates subjected to a concentrated load at mid-span. The derivation provided a framework for the analytical approach of plated beams. However, it neglected the contribution of vertical displacement to the adhesive shear strain thereby under-estimating the shear stress for heavily loaded conditions. In addition, moment equilibrium was not satisfied since the beam axial force was omitted.

Täljsten [10] corrected the shear stress governing equation by considering the beam axial force and provided a solution for an arbitrarily located concentrated load within the beam span. A comparison with finite element analysis (FEA) was also made to validate his model. Malek et al. [11] formulated the model more rigorously by considering the contribution of vertical displacement of the beam to the shear strain provided a solution for an arbitrary distribution of moment by writing the moment function as a quadratic polynomial.

Rabinovich et al. [12] provided a higher-order solution, which accounts for the variation of peel and shear stresses through the thickness of the adhesive layer and satisfies the zero shear stress condition in the adhesive at the plate ends, which is violated in first-order analyses. Similarly, Shen et al. [13] accounted for the longitudinal stresses in the

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Notations

α_b	coefficient of thermal expansion of beam
α_p	coefficient of thermal expansion of plate
β	characteristic value of the governing equation of peel solution
γ_a	shear strain in adhesive
γ_{ad}	shear strain in adhesive in plastic region
γ_e	shear yield strain of adhesive
γ_p	plastic shear strain in adhesive
$\gamma_{\max}$	maximum shear strain in adhesive
ε_a	normal strain in adhesive
ε_b	axial strain at the neutral axis of beam
$\varepsilon_{\mathrm{ba}}$	axial strain at beam/adhesive interface
ε_p	axial strain at the neutral axis of plate
$\varepsilon_{\mathrm{pa}}$	axial strain at plate/adhesive interface
$\Delta\varepsilon_{\mathrm{pb}}$	lack-of-fit strain
$\Delta\varepsilon_0,\Delta\varepsilon_1,\Delta\varepsilon_2$	coefficients of lack-of-fit strain
λ	characteristic value of the governing equation of shear solution
ν	Poisson's ratio of adhesive
ξ	horizontal coordinate in elastic region
σ_a	peel stress in adhesive
$\sigma_{\max}$	maximum peel stress in adhesive
τ_a	shear stress in adhesive
τ_y	shear yield strength of adhesive
ψ_b	curvature of beam
ψ_p	curvature of plate
$\Delta\psi_{\mathrm{pb}}$	lack-of-fit curvature
$\Delta\psi_0,\Delta\psi_1,\Delta\psi_2$	coefficients of lack-of-fit curvature
A_b	cross-sectional area of beam
A_p	cross-sectional area of plate
a_1,a_2,a_3	constants in the governing equation of peel solution
b	width of plate
C_1,C_2,C_3,C_4	constants in the shear solution
D_1,D_2,D_3,D_4	constants in the peel solution
d	plastic zone size
E_a	modulus of adhesive
E_a'	apparent modulus of adhesive
E_b	tensile elastic modulus of beam
E_p	tensile elastic modulus of plate
f_1,f_2	constants in the governing equation of shear solution
G_a	shear modulus of adhesive
I_b	moment of inertial of beam
I_p	moment of inertia of plate
L	length of beam
l	length of plate
M	bending moment due to applied loads
M_b	bending moment in beam
M_p	bending moment in plate
M_p^*	transformed bending moment in plate
M_{PS}^*	particular solution of the transformed bending moment of plate
N_b	axial force in beam
N_p	axial force in plate
N_{pd}	axial force in plate in the plastic region
N_{PS}	particular solution of the axial force of plate
ΔT	change of temperature
t_a	thickness of adhesive layer
U_S	maximum shear strain energy
u	horizontal displacement of adhesive
u_b	horizontal displacement at the centroid of beam
u_{ba}	horizontal displacement of beam/adhesive interface
u_p	horizontal displacement at the centroid of plate
u_{pa}	horizontal displacement of plate/adhesive interface
V_p	shear force in plate
v	vertical displacement of adhesive
v_b	vertical displacement of beam
v_p	vertical displacement of plate
x	horizontal coordinate measured from the plate end
y	vertical coordinate measured from the plate end
y_b	distance from beam centroid to beam/adhesive interface
y_p	distance from plate centroid to plate/adhesive interface
z	distance between the neutral axes of beam and plate

adhesive layer thereby allowing the shear stresses to vary through the thickness of the adhesive.

Smith and Teng [14], reviewed the analytical models for bond stresses in plated beams and proposed a more accurate model by considering the terms that were omitted in previous models. They further conducted a finite element analysis (FEA) to assess the influence of the simplifying assumptions that were incorporated in their models. They noted that incorporating higher-order terms (or violating the zero shear stress condition near the plate end) only influences the solution in a very small zone near the end of the plate. Given that higher-order models are much more complex, they concluded that first-order models are appropriate for design purposes and provide relatively good descriptions of the stress profiles for analysis. As a development of Smith and Teng [14] and Denton [15], Deng et al. [16] developed a model to predict the bond stresses that are induced by both mechanical and thermal loadings. A finite difference method was adopted to solve for bond stresses in beams strengthened with CFRP plates with tapered ends. The tapered geometries help to reduce the bond stress concentrations near the plate ends.

Stratford and Cadei [17] proposed an analytical model that was derived directly from shear and through thickness compatibility, which incorporates mechanical and thermal loadings, pre-stressing of the CFRP, mechanical clamping, and complex plate geometry. This model introduces a lack-of-fit strain and curvature across the adhesive joint, which is assumed equal to the mismatch of strain and curvature across the interface without the presence of the adhesive. This assumption greatly simplifies the solution process. This model has been adopted by current design guidelines [18,19].

Narayanamurthy et al. [20] proposed a model that adopts the superposition principle to generalize the solution of bond stresses in an arbitrary loading arrangement. Zhang et al. [21] developed a solution for curved beams strengthened with bonded plates. Narayanamurthy et al. [22] improved their previous model by considering the shear deformation of the adherends using Timoshenko's beam theory. The effects of shear deformations of the beam on the interfacial shear and peel stresses were quantified, which may be significant when analyzing beams with very short spans.

Based on previous studies of interfacial stresses of beams strengthened with pre-stressed plates [23,24], Ghafoori and Motavalli [25] derived a new closed-form solution by considering the effect of shear deformation of the beam. A parametric study was conducted to study the effects of different strengthening parameters, and a comparison with experimental results was made to validate the model. Experimental results show that first order elastic solutions can give a relatively good prediction for the deboning loads of plated steel beams [8], since commercial structural adhesives usually allow a small amount of localized stress redistribution which somewhat mitigates the effect of bond stress concentrations. For such cases, a first order stress-based solution averaging the stresses through the thickness of the adhesive layer may be applicable. However, for cases with thicker adhesive layers, where the non-uniformity through the thickness of adhesive cannot be neglected; or for rather brittle adhesive for which the stress