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Complete generalization of the Ayrton-Perry formula for beam-column buckling problems

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ABSTRACT

The Ayrton-Perry (or Perry-Robertson) formula based stability resistance model (APF) is very popular in steel structural design standards. Although the original version of the model is more than 100 years old, it is still frequently used and continuously researched due to its simplicity and adaptability. The original and most widely accepted version of the APF is valid only for the flexural buckling of compression members yielding the basic formulation of the column buckling curves of several structural design codes. Recently there were more successful attempts for the extension of the APF type resistance model for other buckling modes such as torsional buckling or lateral-torsional buckling. The paper continues this research by deriving a complete closed-form universal APF type solution for steel beam-column stability problems. Rigorous mathematical solution is given for the so-called "fundamental case" which is defined by a simply supported prismatic beam-column with arbitrary cross-section subjected to uniform compression and biaxial bending. The exact interpretation and the universal form of the member slenderness, imperfection and reduction factors are presented for all possible buckling cases. The results of the paper can widen significantly the field of applicability of APF based design methods providing a theoretically consistent physical model for the beam-column stability problems.

1. Introduction

This paper discusses the theoretically exact derivation and the consistent generalized forms of the Ayrton-Perry formula (APF) for various steel beam-column stability problems. The original APF analytically defines the load carrying capacity of a geometrically imperfect column subjected to pure compression. The formula is based on the onset of yielding in the most compressed fibre calculated from the elastic second order member forces [1]. This simple analytical model is very suitable to describe the complex mechanical behavior of the member buckling phenomenon and to calibrate conveniently to experimental or high level numerical results, as it is demonstrated in the next section. Accordingly the APF has been adopted by several modern structural standards as the basic design model for the buckling resistance of steel members [2–5]. The first application for the purpose of standardized structural design is dated back to the early British code [6] in 1932, where supported by the experimental results of Robertson [7] the APF was introduced for the design of compressed members. Later, after the wide experimental and numerical program on column buckling the European buckling curves were established by Beer and Schulz [8] and finally the APF was used to model the multiple design buckling curves of ECCS. The basic advantage of the APF is that it can be very accurately calibrated to the experimental buckling curves by one parameter only: the imperfection factor [9]. Up to this time the APF had been used only for the modelling of flexural buckling of compression members, the fundamental problem it had been originally developed for. In 1991 it was the first time that the APF based model was proposed for a different buckling problem: in [10] it was extended for lateral-torsional buckling. However it should be noted that the authors in [10] failed to consistently derive the APF for the lateral-torsional buckling problem, thus the original form of column buckling was used and calibrated to experimental results. It is also important to see that the ECCS column buckling curves – which were established almost 40 years ago – are still unchanged and valid in the Eurocode 3 [2] and in many other design codes and proved to be accurate according to several investigations during this time. The standard design model for lateral-torsional buckling has however been modified in the later version of the Eurocode 3 [2] and is still under research due to lack of proper consistency and accuracy. The main reason for this is the lack of theoretical derivation behind the valid design formula, which reveals the significance of a consistent mechanical background for the proper buckling problem. This was recognized by Chapman and Buhagiar who first derived the theoretically consistent APF for different problem than flexural buckling of compressed members: it was the torsional and flexural-torsional buckling of thin-walled members subjected to compression [11]. Later Szalai and Papp developed the theoretically

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Nomenclature

Matrix-vector notations

$\boldsymbol{A}$	linear operator representing the second order stiffness matrix
$\boldsymbol{A}_{alt}$	alternative form of $\boldsymbol{A}$ for doubly symmetric cross-sections
$\boldsymbol{D}$	linear differential operator representing the first order stiffness matrix
$\boldsymbol{D}_0$	zero order part of $\boldsymbol{D}$
$\boldsymbol{D}_2$	second order part of $\boldsymbol{D}$
$\boldsymbol{F}$	external force vector
$\boldsymbol{R}$	resistance force vector
$\boldsymbol{S}$	internal force vector
$\boldsymbol{S}^{I}_{Active}$	active part of the first order internal force vector
$\boldsymbol{S}^{I}_{passive}$	passive part of the first order internal force vector
$\Delta\boldsymbol{S}^{II}_{Load}$	second order internal force vector increment from applied loads only
$\Delta\boldsymbol{S}^{II}_{Imp}$	second order internal force vector increment from imperfection only
$\boldsymbol{U}$	displacement vector
$\boldsymbol{U}_1$	second order displacement vector due to the applied loads only
$\boldsymbol{U}_2$	second order displacement vector increment due to the imperfection only
$\boldsymbol{U}_2^{tot}$	total second order displacement vector due to the imperfection only ($\boldsymbol{U}_0 + \boldsymbol{U}_2$)
$\boldsymbol{U}_0$	geometrical imperfection vector identical to a buckling mode of the system

Roman letters

A	cross-section area
B	internal bimoment
B_{sec}	bimoment resistance of the cross-section
E	elastic modulus
e_0	lateral deflection of the compressed flange as geometric imperfection component
f_y	yield stress
G	shear modulus
h	height of the doubly-symmetric cross-section
I_y, I_z	second moment of inertia about the strong and weak axis
I_w	warping moment of inertia
I_t	St. Venant torsional constant
M_y,M_z	external or internal strong and weak axis bending moment
$M_{y,sec}$,$M_{z,sec}$	strong and weak axis bending moment resistance of the cross-section
M_{cr}	elastic critical bending moment for pure lateral-torsional buckling
N	external or internal compression force
N_{sec}	compression force resistance of the cross-section
$N_{cr,x}$	elastic critical compression force for pure torsional buckling
$N_{cr,y}$	elastic critical compression force for pure strong axis flexural buckling
$N_{cr,z}$	elastic critical compression force for pure weak axis flexural buckling
r_0	radius of gyration
x,y,z	longitudinal centroidal, strong and weak axis of the member
y_0,z_0	position of the shear center in the principal centroidal system
u,v,w	longitudinal, strong and weak axis displacement component
u_0,v_0,w_0	longitudinal, strong and weak axis geometrical imperfection component
W_y, W_z	section moduli about the strong and weak axis (can be elastic or plastic)
W_w	warping section moduli (can be elastic or plastic)

Greek letters

α_{cr}	elastic critical load multiplication factor
α_{sec}	first order cross-section capacity load multiplication factor
$\alpha_{sec,a}$,$\alpha_{sec,p}$	first order cross-section capacity load multiplication factor for the active and passive loads respectively
$\alpha_{sec,N}$,$\alpha_{sec,My}$,$\alpha_{sec,Mz}$	first order cross-section capacity load multiplication factor for the pure compression, strong and weak axis bending
$\overline{\alpha}_{sec}$	modified cross-section capacity load multiplication factor
α_b	buckling resistance load multiplication factor
β_y,β_z	parameters of monosymmetry for the strong axis and weak axis asymmetry respectively
η	generalized imperfection factor
$\overline{\eta}$	alternative form of the generalized imperfection factor
λ	generalized slenderness factor
$\overline{\lambda}_z$	generalized slenderness factor for weak axis flexural buckling
$\overline{\lambda}_y$	generalized slenderness factor for strong axis flexural buckling
$\overline{\lambda}_{LT}$	generalized slenderness factor for lateral-torsional buckling
μ	interaction parameter in the generalized imperfection factor
χ	generalized buckling reduction factor
φ	rotation about the longitudinal axis displacement component
φ_0	rotation about the longitudinal axis geometrical imperfection component

consistent APF for the case of lateral-torsional buckling of beams [12] considering the buckling mode for the shape of the initial geometrical imperfections. In this work the necessary conditions were established for the consistent generalization of the APF for beam-column buckling problems. Based on these findings in [12] the APF was derived for the lateral-torsional buckling of beam-columns subjected to compression and bending considering constant compression effect. Naumes at al. [13] also applied the APF to beam-columns modelling the compressed flange as an equivalent column, and they proposed an approximate solution for non-uniform members and loading as well. Applying a similar approach recently the APF was also used to develop design model for tapered columns [14] and beams [15]. More recently a completely validated and verified new design method was proposed for the APF based resistance calculation of beam-columns [16] subjected to major axis bending and compression.

Extension of the applicability of the APF is consequently useful when creating new design methods for further, specific buckling problems. The main objective of the paper is however to develop a consistent mechanical background for a new buckling design approach based on the overall elastic critical buckling analysis of complete structural models [17]. This new design approach is referred to as Overall Stability Design Method (OSDM) in this paper. The basic idea of OSDM is that it no longer separates the pure loading and buckling modes of the generally loaded members – for instance the compression and bending corresponding to flexural and lateral-torsional buckling modes – but considers the complex loads and forces evaluating the