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Reliability-based design optimization and uncertainty quantification for optimal conditions of composite structures with non-linear behavior

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ABSTRACT

An approach to reliability-based design (RBDO) of beam reinforced composite structures with non-linear geometric behavior is proposed. A unified approach following both *buckling* and *first-ply failure* (FPF) is used to verify the integrity of beam reinforced shallow shell laminated structures. A new RBDO methodology using a genetic algorithm and a hierarchical decomposition searches the global most probable failure point (MPP). For the reliability analysis, the random parameters are the mechanical properties of laminates. Simultaneously the optimal design based on weight minimization under prescribed reliability and buckling constraints is searched through this hierarchical genetic algorithm (HGA). The design variables are the ply angle, the ply thickness, the height and the width of the cross sections of the stiffeners. Numerical results show the capabilities of the proposed approach using the MPP search inner loop integrated in a HGA scheme. Based on a sensitivity methodology the uncertainty for the optimal solution obtained from HGA is analyzed. In the neighborhood of critical buckling values of the structural response the asymptotic behavior of uncertainty propagation is observed. The influence of uncertainties from random parameters and design variables are studied on critical load factor and critical displacement. The variability of these structural response functions are measured by their coefficients of variation and Sobol indices. The most important influences for uncertainty propagation are obtained from ply angle of shell laminates and from longitudinal elastic modulus group.

1. Introduction

The increasing applications of composite materials need to incorporate uncertainties in design tool. Indeed, the manufacture of products made of composite materials often involves a balance between the controllable factors such as component sizing and uncontrollable aspects such as material and processing variations. Variability in the performance of composite materials arises mainly from the variability in mechanical properties, fibre distribution, structural geometry, loading conditions and also manufacturing process. Since composite materials exhibit a complex relationship between the referred parameters and structural response measures this variability can lead to a catastrophic failure [1]. The Reliability-Based Design Optimization (RBDO) covers optimal design problems under given constraints of catastrophic failure probability in rare extreme events. Indeed, most of constructive codes (such Eurocode) impose a lower limit for reliability index, $\beta = 3$, which corresponds to a very low probability of failure (for a Normal probability distribution function (PDF), probability of failure close to 0.0013). The meaning of reliability in structural applications involves the failure analysis under uncertainty in structural design variables, parameters, mechanical properties of materials, loads, etc. This interpretation corresponds to a probabilistic analysis of failure. The failure envelope changes due to uncertainty are also considered in the analysis using the reliability analysis methods in RBDO. Furthermore, the potential for failure due to uncertainty in composite structures is high due to uncontrolled manufacturing parameters, low quality of products, etc. These conditions are challenges for the RBDO applied to composite structures.

The reliability analysis associated with optimal design applied to composite structures has increased in the last twenty years and Reliability-Based Design Optimization (RBDO) of composite structures is currently a very important area of research (Boyer et al. [2]; Conceição António [3,4]; Zheng Y, Das [5]; Adali et al. [6]; Rais-Rohani and Singh [7]; Carbillet et al. [8]; Eamon and Rais-Rohani [9]; Conceição António and Hoffbauer [10]; Salas and Venkataraman [11]; Kriegesmann et al. [12]; Luo et al. [13]). A recent review on current developments on reliability analysis and RBDO applied to composite structures can be found in Chiachio et al. [1].

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The structural integrity analysis of composite structures based on probabilistic concepts is very expensive from the computational point of view. The problem is exacerbated by convergence difficulties associated to non-linearity and dimension of the structures. In particular, buckling is a nonlinear phenomenon with high dependency on geometry as well as stiffness and boundary conditions. Buckling of shell structures can be sensitive to imperfections that are associated to structural failures at loads lower than the designed collapse load. Furthermore, few researchers [14,15] have studied the problem of multiple most probable failure points (MPPs) of the limit state functions. However, the problem is equivalent to find local minima. A strategy to identify the global MPP is proposed in this work based on a HGA approach where reliability constrained weight/cost minimization is performed simultaneously with the MPP search.

Considering a non-linear behavior and using a displacement finite element discretization, the equilibrium equations are derived from the Total Lagrangean formulation. Using an approach based on *critical load factor* concept the structural integrity is checked. A unified approach is proposed following both *buckling* and *first-ply failure* (FPF) analysis of beam reinforced shallow shell laminated structures. Then the optimization of geometrically non-linear structures made of beam reinforced composite laminates can be addressed as a weight/cost minimization problem under safety constraints. To reduce the computational costs the MPP search is performed introducing isolation and migration stages. The validity of the proposed formulation and the efficiency of the adopted numerical procedure are addressed with a selected example. Indeed, the constraints of the weight minimization problem defined for the structural reliability are satisfied and the corresponding MPP is obtained.

The paper is organized as follows: A brief reliability assessment review is given in Section 2. In Section 3 the composite shell structures modeling description and the structural response analysis are outlined. The RBDO problem formulation for composite shell structures and the proposed HGA evolutionary algorithm are presented in Section 4 and Section 5. The uncertainty propagation and the sensitivity analysis for the optimal solutions are described in Section 6. The computational results and the discussion are presented in Section 7 and the conclusions are established in Section 8.

2. Reliability assessment

Let $\pi(\pi_1, \pi_2, ..., \pi_n)$ be the vector of basic uncorrelated random variables involved in structural reliability analysis of composite structures with non-linear geometric behavior. Their mean values and variances describe their statistical nature. If the boundary surface of the safety domain is written as

$$z = \phi(\pi_1, \pi_2, ..., \pi_n) = 0 \tag{1}$$

the values of π belonging to the failure domain will satisfy the inequality:

$$z = \phi(\pi) < 0 \tag{2}$$

The probability of failure is defined as

$$P_f = P[\phi(\pi) < 0] = \int_\Omega f(\pi) d\Omega \tag{3}$$

where $f(\pi)$ is the joint probability density function of π, Ω is the failure region related to the so-called limit state function $\phi(\pi)$ separating the design space into failure ($\phi(\pi) < 0$) and safe regions ($\phi(\pi) > 0$). The distribution of the considered basic variables π_i and the considered limit state surface $\phi(\pi)$ are known and the probability of failure can be employed as a measure of reliability. However, the integral in Eq. (3) cannot be evaluated analytically for realistic structures. To overcome this difficulty the moment reliability theory is used in this work namely the so-called Hasofer-Lind reliability index [16,17]. The advantage of this method is that it is invariant with respect to different failure surface formulations in spaces having the same dimension. The Hasofer-Lind method performs reliability analysis in two steps considering the space of standardized variables. The first step consists of projecting Eq. (1) into the standard normal space through the relation,

$$u_i = \frac{\pi_i - \overline{\pi}_i}{\sigma_{\pi_i}} \tag{4}$$

where $\overline{\pi}_i$ and σ_{π_i} , are the mean value and the standard deviation of the basic variables, respectively. The second step is to calculate, in that space, the minimum distance β from the origin to the transformed limit-state surface

$$\phi(u_1, u_2, ..., u_n) = 0 \tag{5}$$

A design is considered as reliable at the β_a level, prescribed by an appropriated code provision, if $\beta \geq \beta_a$. The geometric interpretation can be given such that the hypersphere of radius β_a and centered at the origin of the axes u_i is required to lie entirely in the transformed safety domain. On the other hand, considering that the probability density in the standard normal space decays exponentially with distance from origin, the point with maximum probability of failure on the limit-state surface is the point of minimum distance from the origin. The search of this point can be formulated as a constrained optimization problem,

$$\text{Minimize: } \beta(\mathbf{u}) = (\mathbf{u}^T\mathbf{u})^{1/2}$$

$$\text{subject to: } \phi(\mathbf{u}) = 0 \tag{6}$$

where $\mathbf{u}$ is the vector of the standardized variables defined through Eq. (4) and the respective solution $\mathbf{u}^*$ is referred in the technical literature as the design point or the most probable failure point (MPP).

3. Response analyses of composite structures

3.1. Non-linear behavior and equilibrium paths

Composite laminated shell structures reinforced with stiffeners are considered in this work as shown in Fig. 1. The composite shell structures are built with N_{sl} shell laminates and reinforced with N_{bl} beam laminates. All shell laminates are symmetric and have the same number of plies, N_p. The shell laminates are defined by the stacking sequence of ply angle/ply thickness. Each j-th shell laminate is defined by i-th ply angle $\theta_{i,j}$ and by i-th ply thickness $\bar{t}_{i,j}$ design variables, grouped in the vectors $\theta = (\theta_{1,1}, \cdots, \theta_{N_p,1}, \cdots, \theta_{1,N_{sl}}, \cdots, \theta_{N_p,N_{sl}})$ and $\bar{\mathbf{t}} = (\bar{t}_{1,1}, \cdots, \bar{t}_{N_p,1}, \cdots, \bar{t}_{1,N_{sl}}, \cdots, \bar{t}_{N_p,N_{sl}})$, respectively. For each k-th beam laminate, the design variables of the cross-sections are the width w_k and the height h_k, grouped in the vectors $\mathbf{h} = (h_1, \cdots, h_{N_{bl}})$ and $\mathbf{w} = (w_1, \cdots, w_{N_{bl}})$, respectively. The non-linear behavior of composite structures is simulated using a displacement formulation of the finite element method. In particular, structures are modeled using the Marguerre shell element and a Timoshenko beam element previously described in Conceição António [18]. The Marguerre finite element is specially used in the structural analysis of shallow shells composite applications. In this case, this element is connected with the Timoshenko beam aiming to enable the analysis of shell structures with stiffeners. This kind of structures is very common in composite materials used in naval and aeronautical applications.

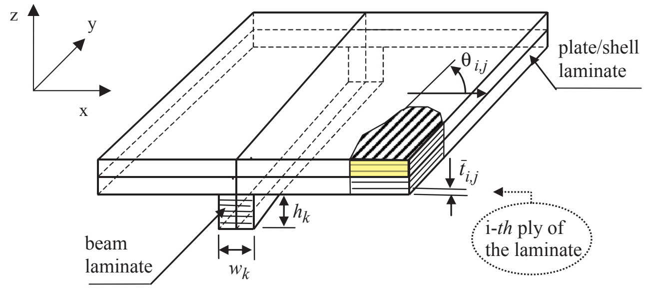

Fig. 1. Composite laminated shell structures reinforced with stiffeners.