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Closed-form approximation of the axial force-bending moment interaction diagram for hollow circular reinforced concrete cross-sections

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ABSTRACT

This paper proposes a simple closed-form approximation of the axial force-bending moment interaction diagram for solid and hollow circular reinforced concrete (RC) cross-sections with an arbitrary number of layers of longitudinal reinforcement. The implemented nonlinear material models are those of the European building code. Specifically, the parabola-rectangular strain-stress relation and the stress-block model are considered for the concrete in compression. On the other hand, the elastic-perfectly plastic behavior is assumed for the reinforcing steel bars. The proposed semi-exact closed-form approximation involves one parameter only, which is calibrated by means of a numerical optimization procedure taking into account a large database of cross-sections. Moreover, an efficient procedure is developed for the design of the outer layer of longitudinal reinforcement in solid or hollow circular RC cross-sections under combined axial force and bending moment (the mechanical ratio of any other layers of longitudinal reinforcement is considered as known function of the mechanical ratio of the outer layer of longitudinal reinforcement). Finally, several numerical examples are included to demonstrate the accuracy of the approximated axial force-bending moment interaction diagram and the effectiveness of the derived design procedure.

1. Introduction

The combined action of axial force and bending moment in reinforced concrete (RC) columns can be due to external lateral forces (e.g., seismic loads, wind actions, liquid and earth pressure), vertical misalignments or unbalanced moments at connecting beams. A common approach to cope with the analysis and design of RC columns under such loading condition is based on the use of the corresponding interaction diagram, which is calculated by considering the ultimate deformed configuration of the cross-section for different positions of the neutral axis. Once proper constitutive laws are assumed for concrete and steel, axial force capacity N_{Rd} and bending moment capacity M_{Rd} are evaluated by imposing equilibrium and compatibility conditions for each position of the neutral axis. The locus of points (N_{Rd},M_{Rd}) identifies the axial force-bending moment interaction diagram of the section. Indeed, a section subjected to the combined action of axial force N_{Ed} and bending moment M_{Ed} can be verified by determining whether the point corresponding to the structural demand (N_{Ed},M_{Ed}) lies inside (safe condition) or outside (unsafe condition) the interaction diagram. Several computer-aided numerical procedures can be adopted to estimate the interaction diagram, see for example Refs. [1–7] for a survey about some recent studies and proposals in this field. Nonetheless, closed-form formulations as well as charts and tables are still valuable tools for use in practice. For instance, closed-form formulations can alleviate the computational effort required for the optimum design of structures or reliability analyses, especially during preliminary stages when a very large number of columns with different characteristics is examined [8]. Papanikolaou and Sextos [9] listed further valid reasons, e.g. end-user verification of the results carried out from commercial software (which are too often used in a black-box sense by practitioners) or educational purposes (by providing immediate results in learning environment).

As regard the shape of the cross-section, the circular one is widely adopted in structural, geotechnical and hydraulic engineering applications because of architectural reasons and/or the strength-invariance with respect to the loading direction [10], see for instance Refs. [11–16]. Despite the widespread use, however, there are few studies and code provisions about circular RC cross-sections compared to the rectangular ones. Within this framework, a simplified numerical methodology for the assessment of the flexural capacity of solid circular

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RC cross-sections has been proposed in Ref. [17], but it does not yield to a closed-form representation of the axial force-bending moment interaction diagram. This limitation has been recently overcame by Trentadue et al. [18], who presented a closed-form approximation of the axial force-bending moment interaction diagram for solid circular RC cross-sections. The case of hollow circular RC cross-sections has deserved even fewer attentions. This contrast with the fact that RC columns with hollow circular cross-section are frequently adopted to build, for instance, bridge piers, chimneys, elevated tanks, pipelines, offshore platforms. The confinement effect in hollow cross-sections is investigated in Refs. [19–21] while the assessment of the shear capacity has been addressed in Refs. [22–25].

The present paper contributes to the analysis and design of circular RC cross-sections by proposing a closed-form approximation of the corresponding axial force-bending moment interaction diagram. The proposed formulation can be applied to solid and hollow circular RC sections with an arbitrary number of layers of longitudinal reinforcement. The selected nonlinear material models are those recommended by the European building code. Specifically, the parabola-rectangular strain-stress relation and the stress-block model are assumed for the concrete in compression whereas the elastic-perfectly plastic behavior is adopted for the reinforcing steel bars. The proposed approximation involves a single parameter that has been calibrated numerically using a large database of reference cross-sections. A simple iterative but derivative-free design procedure has been also developed for the rapid design of the mechanical ratio of the outer layer of longitudinal reinforcement (the mechanical ratio of any other layers of longitudinal reinforcement is considered as known function of the mechanical ratio of the outer layer of longitudinal reinforcement). Some numerical applications are finally illustrated to validate the approximated axial force-bending moment interaction diagram and the proposed design procedure.

2. Geometry and materials

The geometry of a hollow circular RC cross-section is shown in Fig. 1, together with the stresses of concrete and reinforcing bars for a generic position of the neutral axis (compressive strains, stresses and forces are negative). Outer radius and inner radius of the concrete section are indicated as R_{ce} and R_{ci}, respectively. For what follows, it is useful to assume $R_{ci} = \rho_c R_{ce}$ with $0 \leqslant \rho_c < 1$ (a solid cross-section is obtained for $\rho_c = 0$). An arbitrary number N_s of layers of longitudinal bars is considered. The reinforcing steel bars of the jth layer are placed at a constant distance $R_{s,j}$ (with $\rho_c R_{ce} \leqslant R_{s,j} < R_{ce}$) from the center of the section. The total number of steel bars belonging to the jth layer of longitudinal reinforcement is $n_{s,j}$. Each reinforcing bar of the jth layer has radius $r_{s,j}$ and area $A_{s,j}$. The jth layer of longitudinal reinforcement is replaced with a thin steel tube whose area is equivalent to the total area of its reinforcing bars, i.e. $2\pi R_{s,j} t_{s,j} = n_{s,j} A_{s,j}$.

Two equivalent stress-strain relations are considered to model the behavior of the concrete in compression while its tensile strength is neglected. The first model for the concrete in compression is the parabola-rectangular strain-stress relation. It is widely adopted in several design codes, see for instance Refs. [26,27]. Let $|\varepsilon_c|$ be the absolute value of the concrete compressive strain, the corresponding compressive stress $-\sigma_{cd}$ is then evaluated as follows:

$$-\sigma_{cd} = \begin{cases} -f_{cd}\left[1-\left(1-\frac{|\varepsilon_{cd}|}{\varepsilon_{c2}}\right)^n\right] & \text{if } 0 \leqslant |\varepsilon_{cd}| \leqslant \varepsilon_{c2} \\ -f_{cd} & \text{if } \varepsilon_{c2} < |\varepsilon_{cd}| \leqslant \varepsilon_{cu} \end{cases}, \tag{1}$$

where f_{cd} is the design cylindrical strength of the concrete whereas ε_{c2} and ε_{cu} are limit deformation of the parabolic strain-stress relation and ultimate compressive strain, respectively. The parameter n will be assumed equal to 2. According to Eurocode 2 [26], $n = 2$ for normal strength concrete (i.e., concretes having a cylindrical characteristic strength f_{ck} less than or equal to 50 MPa). Moreover, the parameter μ_c is introduced, with $\mu_c = \varepsilon_{cu}/\varepsilon_{c2}$. Several design codes also include a simplified stress-block to model the concrete response under compression [26,27]. Besides Eq. (1), therefore, the compressive stress $-\sigma_{cd}$ can also be evaluated as follows:

$$-\sigma_{cd} = \begin{cases} 0 & \text{if } 0 \leqslant |\varepsilon_{cd}| < \varepsilon_{c4} \\ -f_{cd} & \text{if } \varepsilon_{c4} \leqslant |\varepsilon_{cd}| \leqslant \varepsilon_{cu} \end{cases}. \tag{2}$$

In this case, the parameter μ_c is defined as $\mu_c = \varepsilon_{cu}/\varepsilon_{c4}$. When using Eq. (1) or Eq. (2), it will be also assumed $\mu_c > 1$.

As usually done in analytical capacity models, the concrete is supposed homogeneous within the cross-section in order to derive compact formulations. It should be pointed out, however, that concrete properties can vary within solid and hollow cross-sections. For instance, concrete strength variation in the wall of cylindrical spun-cast concrete elements has been investigated in Ref. [28]. Moreover, it is well known that confined concrete exhibits enhanced strength and ductility. A straightforward but conservative approach to take into account the confinement effects under the hypothesis of homogeneous concrete

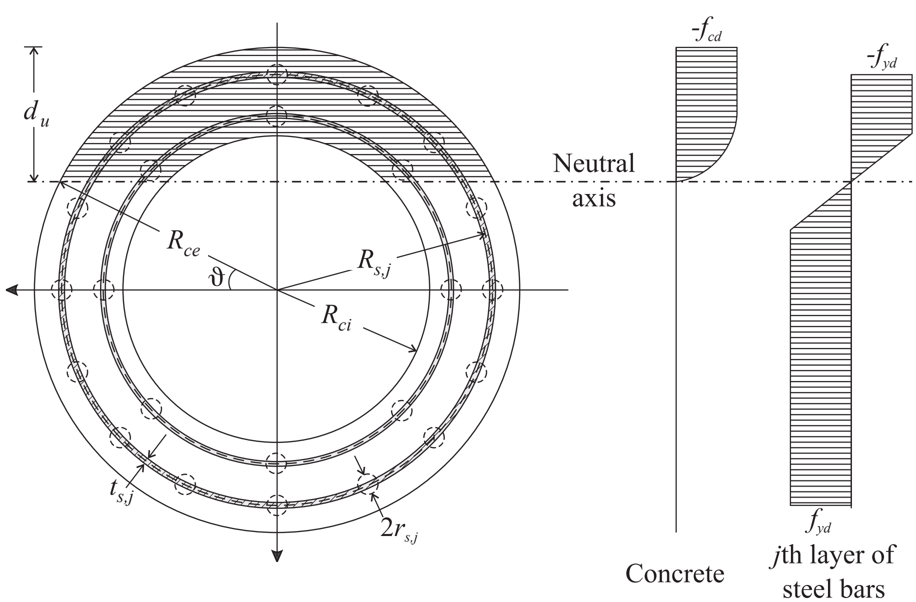

Fig. 1. Hollow circular RC cross-section: geometry and stresses (the parabola-rectangular model is assumed for the strain–stress relation of the concrete in compression).