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Nonlinear static and transient isogeometric analysis of functionally graded microplates based on the modified strain gradient theory

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The objective of this study is to develop an effective numerical model within the framework of an isogeometric analysis (IGA) to investigate the geometrically nonlinear responses of functionally graded (FG) microplates subjected to static and dynamic loadings. The size effect is captured based on the modified strain gradient theory with three length scale parameters. The third-order shear deformation plate theory is adopted to represent the kinematics of plates, while the geometric nonlinearity is accounted based on the von Kármán assumption. Moreover, the variations of material phrases through the plate thickness follow the rule of mixture. By using Hamilton's principle, the governing equation of motion is derived and then discretized based on the IGA technique, which tailors the non-uniform rational B-splines (NURBS) basis functions as interpolation functions to fulfil the C^2-continuity requirement. The nonlinear equations are solved by the Newmark's time integration scheme with Newton-Raphson iterative procedure. Various examples are also presented to study the influences of size effect, material variations, boundary conditions and shear deformation on the nonlinear behaviour of FG microplates.

1. Introduction

In recent years, there has been a considerable increase in research and applications of functionally graded materials (FGMs) in various engineering fields. FGMs are categorized as a class of composite materials [1] since they are constituted from two or more phrases of distinct materials. Those constituent materials in FGMs are varied intentionally and continuously through a prescribed dimension, and hence there is no stress concentration as observed in conventional laminated composites. Ceramic and metal constituents are the most common material phrases from which FGMs are commonly made. In general, the ceramic constituent has a strong capability to withstand a high-temperature effect, whereas the metal counterpart is able to exhibit robust mechanical properties due to its ductility. By combining those constituents with smooth variations of their volume fractions, the preferable mechanical characteristics of both materials are obtained in a unique structure. Thanks to this distinguishing feature, it is no doubt that FGMs have also been studied for applications in cutting-edge devices [2] in which microbeams and microplates are fundamental components. In the mechanical point of view, the behaviour of such microstructures is considerably influenced by the size effect as indicated in various experimental investigations [3–5]. In addition, it was pointed out that the classical elasticity theory is incapable of predicting accurately the responses of the small-scale structures. This is due to the fact that the classical elasticity theory lacks a so-called length scale parameter, which is used to capture the size effect. To deal with this shortage, a number of non-classical continuum theories were proposed in the open literature, such as the strain gradient theory of Mindlin [6], the nonlocal elasticity theory of Eringen [7], the nonlocal strain gradient theory [8], the modified couple stress theory (MCT) of Yang et al. [9] and the modified strain gradient elasticity theory (MST) of Lam et al. [10]. The adoption of those theories to study the behaviour of small-scale structures could be found in

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various studies on nano/microbeams [11–17] nano/microplates [18–28] or nanoshells [29–31]. A critical review of recent research on the application of nonclassical continuum theories for predicting the size-dependent behaviour of small-scale structures can be also found in [32].

Based on the MST, a number of size-dependent models have been developed to predict the responses of microplates on the basis of various kinematic models, such as classical plate theory [33–37], first-order shear deformation theory [38–43] and higher-order shear deformation theories [44–47]. However, these aforementioned works are limited to analytical or semi-analytical methods, which are only applicable to simple problems with certain geometry and boundary and loading conditions. For example, Wang et al. [33], Sahmani and Ansari [44], Gholami et al. [40], Zhang et al. [41,46] and Akgoz and Civalek [47] employed Navier method to derive analytical solutions of rectangular microplates with simply supported boundary conditions, whilst Mohammadi and Fooladi Mahani [35] and Mohammadi et al. [36] used Levy method to derive analytical solutions of rectangular microplates in which two opposite edges are simply supported and the remaning two edges can have arbitrary boundary conditions. The behaviour of microplates with various boundary conditions were also studied using semi-analytical methods such as the differential quadrature method [38,39,42,45,43] and the extended Kantorovich method [34,37]. For the practical problems with complex geometries, loadings and boundary conditions, the application of analytical methods to solve such problems is impossible due to the mathematical complexity of the MST plate models. Therefore, numerical approaches such as finite element method, finite strip method, Ritz method become the most suitable candidates for solving such problems. However, the adoption of classical and high-order shear deformation theories would pose an obstacle for the traditional finite element method as they require a continuity of interpolation functions over the element boundaries. This difficulty is naturally and efficiently handled by using the IGA technique [48], in which the NURBS basis functions are not only smooth and highly continuous but also able to present exact geometries of some conical objects [49–52].

Although numerical solutions of the MST models have been recently developed using Chebyshev-Ritz method [53], the finite strip method [54] and the IGA method [55], these studies were limited to linear problems (linear bending [55], linear buckling [54] and linear free vibration [53,54]). In fact, the behaviour of microplates could undergo large deformations when heavier loads are imposed. Therefore, the geometrical nonlinearity should be considered in the analyses of microplates. However, no literature has been reported for the nonlinear analysis of FG micropaltes based on the MST except a recent study on post-buckling of microplates conducted by Thai et al. [56]. Therefore, the aim of this paper is to propose an effective numerical approach to predict the geometrically nonlinear responses of FG microplates based on the MST and the IGA approach. The displacement field is based on the third-order shear deformation theory of Reddy [57], while the geometrical nonlinearity is accounted by adopting the von Kármán assumption. Hamilton's principle is utilized to construct the weak form of the equation of motion. In addition, the NURBS basis functions are employed as interpolation functions to satisfy the C^2-continuity requirement in the discretization process. The Newmark's integration scheme in conjunction with Newton-Raphson iterative procedure is adopted for the nonlinear static and dynamic analysis. Verification studies are also performed to prove the accuracy of the present approach. The influences of the size effect, material gradient indices, boundary conditions and thickness ratios on the nonlinear responses of FG microplates are firstly investigated through various parametric studies.

2. Plate formulations

2.1. Material properties of FGMs

As described in Fig. 1, the in-plane coordinates x and y are located in the midplane Ω of the plate having the thickness of h, while the z-axis is normal to the midplane. According to the rule of mixture, the variation of material properties throughout the plate thickness is expressed by

$$P(z) = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + P_m \tag{1}$$

where $P(z)$ is a typical material property, such as Young's modulus $E(z)$, Poisson's ratio $\nu(z)$, density $\rho(z)$. P_c and P_m represent the properties of ceramic and metal surfaces, respectively, and the gradient index n is used to describe the profile of material variation. It can be seen that a single ceramic or metal plate is obtained when the gradient index n is prescribed as 0 or ∞.

2.2. Modified strain gradient theory

Based on the MST proposed by Lam et al. [10], the virtual strain energy stored in an elastic body is expressed as

$$\delta U = \int_V \left(\sigma_{ij}\delta\varepsilon_{ij} + p_i\delta\varsigma_i + \tau^{(1)}_{ijk}\delta\eta^{(1)}_{ijk} + m^s_{ij}\delta\chi^s_{ij}\right)dV \tag{2}$$

where the classical stress and high-order stresses are given as follows

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij}; \quad p_i = 2\mu l_0^2\varsigma_i; \quad \tau^{(1)}_{ijk} = 2\mu l_1^2\eta^{(1)}_{ijk}; \quad m^s_{ij} = 2\mu l_2^2\chi^s_{ij} \tag{3}$$

in which l_0, l_1 and l_2 are the material length scale parameters. λ and μ denote the Lamé constants:

$$\lambda = \frac{\nu E(z)}{[1+\nu(z)][1-2\nu(z)]}; \quad \mu = \frac{E(z)}{2[1+\nu(z)]} \tag{4}$$

The classical strain tensor ε_{ij} and high-order strain gradient tensors, namely the dilatation gradient tensor ζ_i, the deviatoric stretch gradient tensor $\eta^{(1)}_{ijk}$ and the symmetric part of rotation gradient tensor χ^s_{ij}, are given as follows

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i} + u_{m,i}u_{m,j}\right); \tag{5a}$$

$$\varsigma_i = \varepsilon_{mm,i} \tag{5b}$$

$$\eta^{(1)}_{ijk} = \eta^s_{ijk} - \frac{1}{5}\left(\delta_{ij}\eta^s_{mmk} + \delta_{jk}\eta^s_{mml} + \delta_{ki}\eta^s_{mmj}\right);$$
$$\eta^s_{ijk} = \frac{1}{3}\left(u_{i,jk} + u_{j,ki} + u_{k,ij}\right) \tag{5c}$$

$$\chi^s_{ij} = \frac{1}{4}\left(e_{imn}u_{n,mj} + e_{jmn}u_{n,mi}\right) \tag{5d}$$

where u_i denote the components of displacement vector, δ_{ij} and e_{ijk} are the Kronocker delta and permutation symbol, respectively.

2.3. Kinematics

The displacement field according to the third-order shear deformation plate theory [57] is expressed as follows

$$\begin{aligned} u_1 &= u + f(z)\theta_x - g(z)w_{,x} \\ u_2 &= v + f(z)\theta_y - g(z)w_{,y} \\ u_3 &= w \end{aligned} \tag{6}$$