Engineering Structures 125 (2016) 494–503

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Exact finite element formulation for an elastic hybrid beam-column in partial interaction with shear-deformable encasing component

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A R T I C L E I N F O

Article history:
Received 25 September 2015
Revised 26 May 2016
Accepted 8 July 2016

Keywords:
Hybrid beam-column
Slips
Shear-flexibility
Exact stiffness
Finite element

A B S T R A C T

The authors present the exact finite element formulation for partially connected shear-deformable elastic hybrid beam-columns with several embedded sections. Euler-Bernoulli's kinematic assumptions are adopted for the embedded section whereas Timoshenko's kinematic assumptions are considered for the encasing element. The shear connection between the encasing component and the embedded section is modeled through a continuous relationship between the interface shear flow and the corresponding slip. A set of coupled system of differential equations in which the primary variables are the slips and the shear deformation of the encasing cross-section is derived from the governing equations describing the behavior of an elastic shear-deformable hybrid beam-column in partial interaction. This coupled system has been solved in closed-form, and the "exact" stiffness matrix has been derived using the direct stiffness method. The latter has been implemented into a general displacement-based finite element code, and has been used to investigate the behavior of shear-deformable hybrid beams. Three numerical examples have been considered in order to assess the capability of the proposed formulation and to investigate the effect of the shear connection stiffness and span-to-depth ratios on mechanical responses of the beam-columns. It has been found that the transverse displacements are more affected by the shear flexibility than the slips.

1. Introduction

The history of construction industry is often influenced by the innovation of new technology as engineers and researchers strive to increase the safety, economy, and performance of our built environment. Innovative structural solutions have been achieved by combining different materials and/or methods of construction to produce a structure with enhanced strength, stiffness, ductility and fire protection. Steel-concrete composite beams, timber-steel concrete floors, coupled shear walls, sandwich beams, concrete beams externally reinforced with laminates and more recently concrete column reinforced by several embedded steel profiles are all examples of composite/hybrid structures in civil engineering. To develop composite action, the longitudinal shear force transferred at the interface between each component is achieved through two main mechanisms: mechanical interaction provided by the shear connectors; and friction assumed to be proportional to the normal stresses at the interface. The analysis of the composite member is complicated due to the partial transfer of longitudinal shear force at the interface. Over the years, there has been a great deal of research conducted on the subject of elastic two-layer composite beams in partial interaction. The first contribution is commonly attributed to Newmark et al. [1] who investigated the behavior of a two-layer beam considering that both layers are elastic and deform according to the Euler-Bernoulli kinematics. In their paper, a closed-form solution is provided for a simply supported elastic composite beam. Since then, numerous analytical models were developed to study different aspects of the composite behavior of two-layer composite beams under more complicated situations. Several analytical formulations to investigate the behavior of elastic two-layer beam were proposed [2–10]. Significant development beyond that available from Newmark et al.'s paper [1] has been made in [9] by considering Timoshenko's kinematic assumptions for both layers. Besides these analytical works, several numerical models, mostly FE formulations have been developed to investigate the nonlinear behavior of both Bernoulli and Timoshenko two-layer beams in partial interaction [11–22]. Most of the papers on composite beam with interlayer slip is restricted to the case of two layers and multi-layered beams as well as hybrid members reinforced by several embedded sections have received less attention. Chui and Barclay [23] and Schnabl et al. [24] proposed an exact analytical model for the case of three-layer beam where the thickness as well as material of the individual layers

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http://dx.doi.org/10.1016/j.engstruct.2016.07.015
0141-0296/

are arbitrary. Sousa et al. [25] developed an analytical solution for statically determinate multi-layer beams with the assumption that the rotation of the beam cross-section is assumed to be the same even if the shear-flexible components with different shear modulus are considered. The analytical solution is derived from the governing equations consisting of a coupled system of differential equations where the interlayer slips are considered as primary variables. Škec et al. [26] proposed mathematical models and analytical solutions for the analysis of linear elastic Reissner multi-layered beams. The models take into account the interlayer slip and the uplift of the adjacent layers, different material properties, independent transverse shear deformations, and different boundary conditions for each layer. Ranzi [27] proposed two types of displacement-based elements to analyze the locking problems in multi-layer Euler-Bernoulli beams in partial interaction. For classical polynomial shape functions, it is shown that the element with internal node well characterizes the partial interaction behavior of multi-layer beams while the element without internal node suffers from the curvature locking problems.

A formulation based on the exact stiffness matrix offers the possibility of generating a locking-free model. These elements are highly attractive due to their precision, computational efficiency and mesh independency. Heinisuo [28] proposed a finite element formulation using exact stiffness matrix for uniform, straight, linear elastic beams with two faces and one core and with three symmetric faces and two identical cores. Sousa [29] developed the analytical formulation and derived the exact flexibility matrix for both shear-deformable and shear-rigid multi-layer beams in partial interaction with the assumption that both transverse displacement and rotation are the same for all layers.

The purpose of this paper is to present a new exact FE formulation for the analysis of shear-deformable hybrid beam-columns in partial interaction based on the exact stiffness matrix derived from the governing equations of the problem. The features of the formulation presented in this paper are as follows: (i) longitudinal partial interactions between the layers are considered which provide a general description of the stresses and strains in the layers; (ii) shear deformation of the encasing component is considered; (iii) exact stiffness matrix is used which provide accurate and stable results. The present model provides, therefore, an efficient tool for linear elastic analysis of hybrid beam-columns with shear-deformable encasing component, arbitrary support and loading conditions. In contrast with the work of Sousa [29], the present formulation allows the cross-section rotation of the encasing component to be different to that of the embedded elements.

The rest of the paper is organized as follows. In Section 2, the governing field equations for a shear-deformable hybrid beam-columns in partial interaction are presented. The governing equations of the problem are derived in Section 3. In Section 4, the full analytical solution of the coupled differential equations is provided, regardless of the loading and the nature of the boundary conditions (support and end force). The exact expression for the stiffness matrix is deduced for a generic shear-deformable hybrid beam-column element in Section 5. Numerical examples are presented in Section 6 in order to assess the performance of the formulation and to support the conclusions drawn in Section 7.

2. Fundamental equations

The field equations describing the behavior of linear elastic hybrid beam-column with n embedded sections in partial interaction are briefly outlined in this section. All variables subscripted with b belong to the encasing section and those with subscript a belong to the embedded section. Quantities with subscript sc are associated with the longitudinal shear connection. The following assumptions are commonly accepted in the model to be discussed in this paper:

- connected members are made out of elastic, homogenous and isotropic materials;
- the cross-sections of all embedded sections remain plane and orthogonal to beam axis after deformation;
- the cross-section of the encasing component remains plane but not (necessarily) orthogonal to beam axis after deformation;
- relative slip can develop along the interface between the encasing component and embedded section and is considered at the centroid of the embedded section;
- the lateral deflection v is assumed to be the same for all components; and
- discretely located shear connectors are regarded as continuous.

2.1. Equilibrium

The equilibrium equations are derived by considering the free body diagrams of a differential elements dx located at an arbitrary position x in the hybrid beam-column, see Fig. 1. The interface connection between the embedded sections and the encasing component is modeled by continuously distributed spring. The equilibrium conditions result in the following set of equations:

$$\partial N_{a_i} + D_{sc_i} = 0 \tag{1}$$

$$\partial N_b - \sum_{j=1}^{n} D_{sc_j} = 0 \tag{2}$$

$$\partial M_b + T_b + \sum_{j=1}^{n} h_j D_{sc_j} = 0 \tag{3}$$

$$\partial M_{a_i} + T_{a_i} = 0 \tag{4}$$

$$\partial T_b + \sum_{j=1}^{n} \partial T_{a_j} + p_y = 0 \tag{5}$$

where

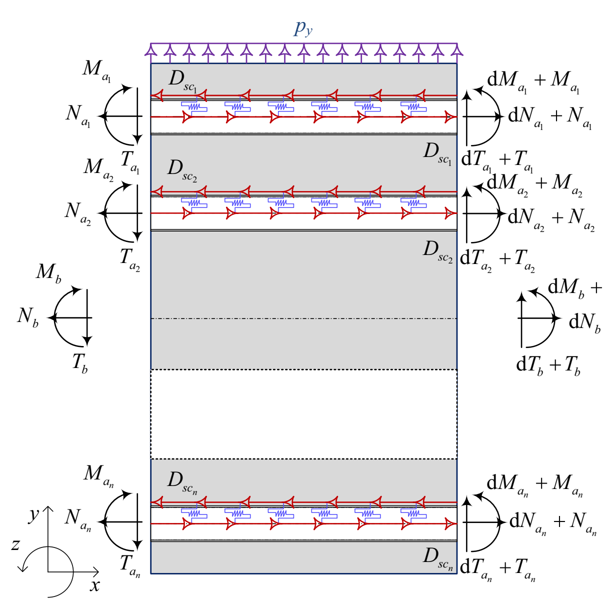

Fig. 1. Equilibrium of a hybrid beam-column.