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Structural optimization of lattice steel transmission towers

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ABSTRACT

This paper presents a methodology for the optimization of three dimensional truss structures and its application to supports of power transmission lines. For that purpose, an efficient modified version of the Simulated Annealing algorithm is developed. The validity of the proposed approach relies on practical and constructional features of the structures considered in this study. A first order sensitivity analysis is implemented to improve the performance of the algorithm and to avoid the computation of the large number of structural analyses usually required by the Simulated Annealing algorithm. The new SA algorithm is successfully applied in two classical benchmark problems and in a real full-scale design problem.

1. Introduction

Trusses are traditionally one of the most used structures in engineering, covering a wide range of solutions for different engineering challenges. There is also a large number of different truss configurations for each purpose. Therefore, the layout or sizing optimization of truss structures have been widely studied by many authors, applying several different methods, in an effort to design the optimum structure under certain conditions. Numerous approaches are focused on sizing optimization [1–7] whose aim is to find the specific bars that lead to the optimum design of a certain fixed geometry; while others face the layout optimization of trusses in a search for the best geometry of the structure even through topology optimization [8–10]. In this paper we define a methodology for combined layout and sizing optimization of truss structures [11–15] and its application to the field of power transmission towers.

The range of optimization methodologies used in this field is really extensive but, in the recent past, metaheuristic algorithms have gained interest and have attracted attention among the optimization community. This family of algorithms offers a great adaptability to a large range of diverse problems, though it is well known they involve tremendous computing requirements, specially when dealing with practical and real engineering problems that involve a large number of variables and constraints.

The common goal in structural optimization is to obtain the structure that, under certain loads and subject to particular conditions, uses the minimum amount of material or, in other words, has the minimum weight. In this study, constrained weight optimization of truss structures is approached including all the particularities of transmission towers. The approach uses continuous variables for the layout optimization and discrete sections for the sizing by selecting elements from an available profile catalog. Multiple constraints (displacement, stress, buckling, slenderness) and load cases are also considered. In this context and given the engineering problem we are facing, we propose a methodology based on the Simulated Annealing algorithm exposed by Kirkpatrick [16] and further studied in [17–29]. A modified and improved version of the Simulated Annealing algorithm is defined and proved effective for the layout and sizing optimization of truss structures. Additionally, a first order sensitivity analysis is implemented to reduce the cost of all the computations the algorithm needs to perform, increasing the efficiency of the method. The method is successfully applied to real power transmission structures.

2. Problem statement

2.1. Objective function

The general structural optimization problem can be written as,

$$\textit{minimize} \quad W = F(\boldsymbol{x}) \tag{1}$$

subject to

$$g_j(\boldsymbol{x}) \leqslant 0, \quad j = 1, \dots, m \tag{2}$$

$$(\boldsymbol{x}_i)_{min} \leqslant \boldsymbol{x}_i \leqslant (\boldsymbol{x}_i)_{max}, \quad i = 1, \dots, n \tag{3}$$

In Eqs. (1)–(3), $\boldsymbol{x}$ represents the vector of n design variables. Eq. (1) corresponds to the objective function to minimize and represents the weight of the structure, g_j in Eq. (2) correspond to the

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m structural constraints considered and inequalities of Eq. (3) indicate the side constraints of the n design variables.

The objective function of the problem we are facing can be expressed in terms of a sum of the weights of each bar or element forming the structure.

$$W = \rho \sum_{i=1}^{n_b} l_i \, A_i \tag{4}$$

where ρ is the density of the material, n_b is the number of bars of the structure and l_i and A_i are the length and cross-sectional area of the i-th element, respectively.

2.2. Design variables

A truss structure is defined by the coordinates of the nodes and the connectivity of the bars. Therefore, there are several different approaches to face their optimization: discrete or continuous sizing optimization, topology optimization, layout optimization of the positions of the nodes of the structure or mixed approaches among others. Many truss structures, and particularly power transmission towers, can be separated in pieces that include a group of certain bars, usually called blocks. The geometry of the tower is then formed joining some of these blocks. The layout optimization can be considered by modifying the dimensions of these pieces, leaving the connectivity fixed. These dimensions will represent the continuous variables of our model.

On the other hand, we can also divide each block in the bars they are formed by. The search for the optimum cross-sectional area for each bar represents the sizing optimization part of the problem. From the practical point of view, it is more appropriate to treat these variables with discrete values since the market usually offers a discrete inventory of cross-sectional areas.

This approach leads to a mixed optimization problem where variables of different natures need to be optimized, which increases the complexity of the problem but also allows to achieve better designs. This aspect is crucial to decide and choose an optimization algorithm since it has to deal not only with discrete variables but with continuous as well, and be able to modify them simultaneously.

2.3. Constraints treatment

The problem stated in (1) is subject to different structural (2) and side (3) constraints. In common engineering problems the number of structural constraints involved is usually large. Although most constraints are devoted to restrain the structural behavior of the structure, they implicate different units of measure as they represent different physical magnitudes. To avoid numerical issues derived from this nature, the structural constraints of the problem are treated in a normalized form.

For each constraint, a ratio is defined between the actual value of the magnitude to analyze and a reference value. Thus, the state of a certain constraint can be easily observed by this ratio; values close to 0 mean a clearly satisfied constraint while values close to 1 mean an active constraint. Values higher than 1 mean violated constraint. It is worth mentioning that a small violation of the constraints is allowed at early stages of the process in order to avoid a fast increment of the number of active constraints that in some cases can lead to a rapid stagnation of the algorithm.

Thereby, the structural constraints indicated in (2) are transformed into:

$$a_j(\boldsymbol{x}) = \frac{\varphi_j(\boldsymbol{x})}{\Psi_j(\boldsymbol{x})} \leqslant 1, \qquad j = 1, \ldots, m \tag{5}$$

where a_j is the normalized constraint, φ is the real value of the analyzed magnitude, Ψ_j is the reference value and m is the number of constraints.

Once the whole optimization problem has been stated, next step is to define the optimization algorithm.

3. Optimization algorithm

We chose the Simulated Annealing algorithm [16–30] for the optimization of lattice towers since it naturally allows the treatment of continuous and discrete variables together. In addition, the Simulated Annealing algorithm is able to avoid possible local minima, which is essential in this case since real application examples involve a large number of constraints and design variables. Its main drawback is the vast number of tests it needs to perform, which is a common handicap in stochastic algorithms and alternative approaches like genetic algorithms. Authors developed a numerical model based on first order Taylor expansions to reduce the computing requirements. Hence, the number of structural analyses required in the optimization process can be drastically reduced.

The algorithm generates a large number of test designs where each one of these tests analyzes a random modification of the present design of the structure using Taylor expansions to reduce computing requirements. Then, each design enters a decision module where it can be accepted or rejected according to the value of the objective function and the fulfillment of the constraints. Every design that violates a constraint is not feasible and it is rejected immediately. When the design is feasible, the decision module can adopt one of the following three possible decisions:

- Downhill move: The new design fulfills all the constraints and reduces the objective function. The new design is directly accepted.
- Uphill move: The new design satisfies the constraints but it involves an increase of the objective function. In this case the probability of acceptance of this design is defined by a Boltzmann–Gibbs probability distribution (P) according to the Metropolis theory [30].

$$P(E,T) = e^{-\Delta E/kT} \tag{6}$$

  where E is the energy function (by analogy, the objective function), ΔE is the difference of energy between the test and the present design, k is the Boltzmann thermal constant and T is a temperature parameter that controls the evolution of the algorithm.

  The decision module of the algorithm generates a random number $\in [0,1]$ with uniform probability distribution function. This number is then compared with (6) as this expression is considered an approximation to the probability of transition between states in the annealing analogy [30]. If the probability $P(E,T)$ is larger than the generated random number then the trial design is accepted even though it increases the objective function.
- Rejected Uphill Move: The new design increases the objective function but does not fit the Boltzmann–Gibbs probability.

Consequently, the natural tendency of the algorithm is to perform downhill movements decreasing the objective function in a search for the optimum design. However, the algorithm also performs certain uphill movements increasing the value of the objective function, apparently moving away from the optimum. This aspect is crucial since it allows the designs to escape from local minima and eventually proceed to the global optimum with new downhill moves.