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# Analytical evaluation of impedances and kinematic response of inclined piles

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A B S T R A C T

This paper presents an analytical model, based on the beam-on-dynamic Winkler foundation approach, for the evaluation of impedances and kinematic response of single inclined piles. The pile is modelled as a Euler–Bernoulli beam having a generic inclination and the soil–pile interaction is captured by defining soil impedances according to expressions available in the literature for viscoelastic layers undergoing harmonic vibrations of a rigid disk. The coupled flexural and axial behaviour of the pile is governed by a system of partial differential equations, with the relevant boundary conditions, that is solved analytically in terms of exponential matrices. The solution for piles embedded in layered soils is achieved according to the direct stiffness approach by using the analytical solutions derived for generic pile sections embedded in homogeneous soils. Expressions of both the soil-foundation impedance functions and the foundation input motion are derived. Some applications, including comparisons of results with those obtained from rigorous boundary element formulations, are performed to evaluate the model capabilities. Classical stiffness and damping coefficients, based on the propagation of shear and pressure waves in plane-strain condition, are used in the applications to account for the soil–pile interaction; anyway, different formulations can be easily implemented.

Results of applications, concerning piles with different inclination in both stiff and soft soils, demonstrate that the model, characterised by a very low computational effort, is able to capture the response of inclined piles subjected to seismic loading. Furthermore, with reference to linear problems, the model allows the derivation of the pile stiffness matrix that can be implemented in commercial computer software based on the finite element approach.

## 1. Introduction

The Beam-on-Dynamic Winkler Foundation approach (BDWF) has been largely used in the literature to study the problem of a single pile subjected to seismic loading. Even if the Winkler's assumption ignores the shear transfer between soil layers, this approach has proven to be an efficient method to solve not only static but also dynamic problems involving the evaluation of the lateral pile response. According to this approach, the soil–pile interaction is captured through the definition of Winkler's coefficients (spring and dashpot coefficients) that may be characterised by a linear or nonlinear behaviour. Concerning nonlinear problems, *p–y* curves, derived from field tests, are typically employed implementing nonlinear force–displacement relationships in which soil–pile gapping, cyclic degradation, and rate dependency can also be included e.g. [1–5]. Despite the nonlinear approach is more suited to model soil–pile interaction phenomena under severe earthquakes, linear (equivalent) approaches are very often preferred for different reasons. The first reason is that they only require few parameters (shear wave velocity, density, damping and degradation curves) to be defined; secondly, they allow the solution in the frequency domain so that the frequency dependent nature of the soil–pile interaction phenomena can be directly included and convergence problems can be avoided; thirdly, they allow the evaluation of the complex Soil–Structure Interaction problem (SSI) according to the substructure approach, studying the problem by separating the kinematic and inertial interaction effects. In this framework, the definition of Winkler's coefficients, able to capture the dynamic stiffness of the soil–pile system and the frequency dependent radiation damping phenomena, constitutes an important task, since they strongly affect both the kinematic and the inertial response of the soil-foundation system [6]. Providing a

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state of the art of the various formulations available in the literature goes beyond the scope of this paper; however, it should be remarked here that major contributions to the definition of these parameters are due to Novak [7], Kagawa and Kraft [8], Gazetas and Dobry [9,10], Kavvadas and Gazetas [11] and, more recently, to Mylonakis [12], Sica et al. [13] and Shadlou and Bhattacharya [14]. In addition, whilst the nonlinear problem is classically solved through the finite element approach, the linear one may be often formulated and solved analytically. By adopting formulations proposed by above mentioned authors, the BDWF approach has been used extensively to study the response of vertical piles subjected to dynamic loading for which simplified expressions for the prediction of the dynamic stiffness and the kinematic pile response have also been derived. On the contrary, applications of such approaches to inclined piles are not so popular in the literature and more sophisticated finite element or boundary element models are preferred. As an example, with reference to inclined pile groups, Gerolymos et al. [15] and Giannakou et al. [16] have recently published results of numerical investigations based on finite element discretization of the soil–pile group system, whilst Padrón et al. [17–19] and Medina et al. [20] have proposed a numerical model by adopting a BEM–FEM coupled formulation for the evaluation of the dynamic stiffness and kinematic response of floating or end-bearing inclined pile groups or single piles. Furthermore, Dezi et al. [21] have developed a finite element model for the kinematic analysis of inclined pile groups exploiting elastodynamic Green's functions proposed by above mentioned authors [7–14] to capture the pile–soil–pile interaction, avoiding directly modelling the soil domain with a significant saving of computational time. The model is also applicable to single vertical or inclined piles.

In this paper an analytical approach for the kinematic interaction analysis of inclined single piles, based on the BDWF approach is presented, and an analytical solution is derived. The pile is modelled as a Euler–Bernoulli beam having a generic inclination and the soil–pile interaction is captured by defining soil impedances according to expressions available in the literature for viscoelastic layers undergoing harmonic vibrations of a rigid disk. The coupled flexural and axial behaviour of the pile is described by a system of partial differential equations, with the relevant boundary conditions, that is solved analytically exploiting exponential matrices. The model allows defining analytical expressions for both the frequency–dependent soil–foundation impedance matrix and the foundation input motion. Applications include comparisons of results, in terms of impedances, kinematic response and stress resultants, with those available in the literature, obtained from rigorous boundary element formulations. The model furnishes an overall good response and is characterised by a very low computational effort compared with that relevant to finite element or boundary element approaches.

## 2. Analytical model

The analytical formulation of the dynamic problem of a single pile embedded in a stratified soil deposit and subjected to the free–field seismic displacements is presented in this section. Each soil stratum is supposed to be homogeneous, namely characterised by constant properties. The problem is formulated in the frequency domain by assuming a linear viscoelastic behaviour for the pile and soil; nevertheless, the soil nonlinearity can be considered using a linear equivalent model. The soil is modelled as independent infinite viscoelastic horizontal layers and the dynamic soil–pile interaction, including the hysteretic and geometric damping, is captured adopting available analytical solutions for a vibrating rigid disk. Such assumptions constitute the basis of BDWFs adopted for vertical piles and, in this paper, are adopted for the case of inclined piles although a more significant coupling between the layers is expected.

The soil–pile interaction within each homogeneous soil stratum is firstly analysed and the solution derived analytically exploiting exponential matrices. Solutions obtained are then used, according to the direct stiffness method, to pursuit the solution in the case of stratified soil.

### 2.1. Kinematics

A single circular pile of diameter $\phi$ and length $L$, embedded with a generic inclination in a horizontally stratified soil deposit and subjected to a free–field motion, is considered (Fig. 1). One homogeneous stratum of the soil deposit and the relevant section of the embedded pile are isolated and a global orthonormal reference system frame $\{0, x, y, z\}$ is defined with the origin at the top plane of the stratum and the $z$ axis directed downward (Fig. 2a).

A local reference system $\{0, \xi, \eta, \zeta\}$ of the pile, having the $\zeta$ axis passing through centroids of the circular cross sections, is also defined; if the orientation of $\zeta$ is given by the unit vector $\mathbf{e}_\zeta$, the orthonormal basis is completed by the two unit vectors

$$\mathbf{e}_\eta = \mathbf{e}_z \times \mathbf{e}_\zeta \quad \mathbf{e}_\xi = \mathbf{e}_\eta \times \mathbf{e}_\zeta \tag{1a, b}$$

so that $\eta$ axis lies on the top horizontal plane whereas $\xi$ axis lies on the vertical plane containing the pile axis. By denoting by

$$c_{ab} = \mathbf{e}_a \cdot \mathbf{e}_b \tag{2}$$

the cosine of the angle formed by two directions $a$ and $b$, the length of the pile section embedded in the stratum is

$$L = \frac{H}{c_{z\zeta}} \tag{3}$$

where $H$ is the thickness of the stratum.

With reference to points lying on the pile axis, if $\omega$ is the circular frequency, the soil displacements at depth $z$ are described by the complex valued vector

$$\mathbf{u}_g(\omega; z) = \begin{bmatrix} \mathbf{u}_h(\omega; z) \\ u_z(\omega; z) \end{bmatrix}_{3\times1} \tag{4}$$

in which sub-vector $\mathbf{u}_h$ groups horizontal components in $x$ and $y$ directions. Similarly, the free-field motion at the pile location $\mathbf{u}_{ff}$ is partitioned by grouping horizontal components in sub-vector $\mathbf{u}_{ff,h}$

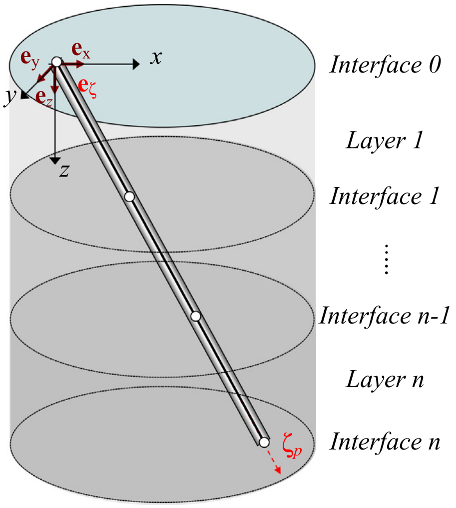

**Fig. 1.** Inclined pile in a generic layered soil deposit.