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A finite-difference formulation of elastic rod for the design of actively bent structures

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ABSTRACT

A discrete formulation of elastic rod has been tailored for the particular design task of geometric modelling, form finding and analysis of actively bent structural systems. The rod element is fully described by using vector based quantities, hence making it easy to implement and be suitable for explicit resolution methods such as the Dynamic Relaxation (DR). From this point of view, the model under consideration aims to provide a natural enhancement, of existing DR schemes of elastic rods, primarily formulated for analysis/design of stressed spline structures with isotropic cross-section, whilst, the proposed formulation allows for the general case of initially straight rods with anisotropic cross-section and torsional stiffness effects, to be taken into consideration. In order to avoid numerical conditioning problems, the method adopts a reduced Degrees of Freedom approach, however, the design limitations usually involved with such an approach, are 'removed' by adopting the Bishop theory of framed curves, hence making it possible to reduce to only three (translations) the Degrees of Freedom to be explicitly computed by numerical integration of the corresponding acceleration terms.

1. Introduction

In the context of Architectural Geometry [1], the development of numerical tools, to assist the design and form exploration of actively bent structures, is gaining increasing interest [2–6]. According to Lienhard et al. [7] the term 'Active-Bending' refers to those design cases in which the structural shape is obtained as a result of bending frameworks/assemblies of elastic members such as (but not limited to) rods or beams. Examples of constructing shelters and huts 'by bending' of branches, sticks or laths, probably date back to prehistoric times. Excepting those episodes of vernacular architecture, as for instance, the iconic mongolian Yurt [8]: aware-driven-designs examples of using bending as a self-forming process for the shape definition of roof structures (for both temporary [9,10] or permanent use [11–13]) are fairly recent. Particularly, in the last few years, an increasing number of experimental pavilions [14–18] have been built around the world, by academics/professionals, in (both) Architecture/Structural Engineering, mostly as a means of drawing attention on such a 'new' method of building 'through' bending.

For the design of such actively bent systems, shape and material aspects are tightly connected through the particular construction (bending) process, meaning that: physical and/or numerical models are required to be used during the design process (see Figs. 1, 2) in order to take the structural mechanical behaviour of the system into account while defining the architectural shape [19].

The discrete mathematical modelling of elastic rods is an expanding research field, finding application in several areas, for instance, in medicine [20], biology [21], computer graphics [22–24], applied physics [25], computer aided design [26,27] and structural engineering [28,29]. Due to the large amount of literature on the topic: rather than providing a long list of previous works, it has been aimed (in the next subsections) to concentrate on the most relevant requirements upon which a model of discrete elastic rod suitable to aid the design of actively bent systems can be built. This will make possible to reduce the number of existing contributions to only few, as those most relevant to our need. In particular: a set of 'main' references throughout the paper is represented by the works of Adriaenssens and Barnes on stressed spline structures [30–32].

1.1. Resolution method: implicit or explicit?

For the physical simulation of elastic rods, and (in general) for every procedure aimed to numerically solve systems of ordinary

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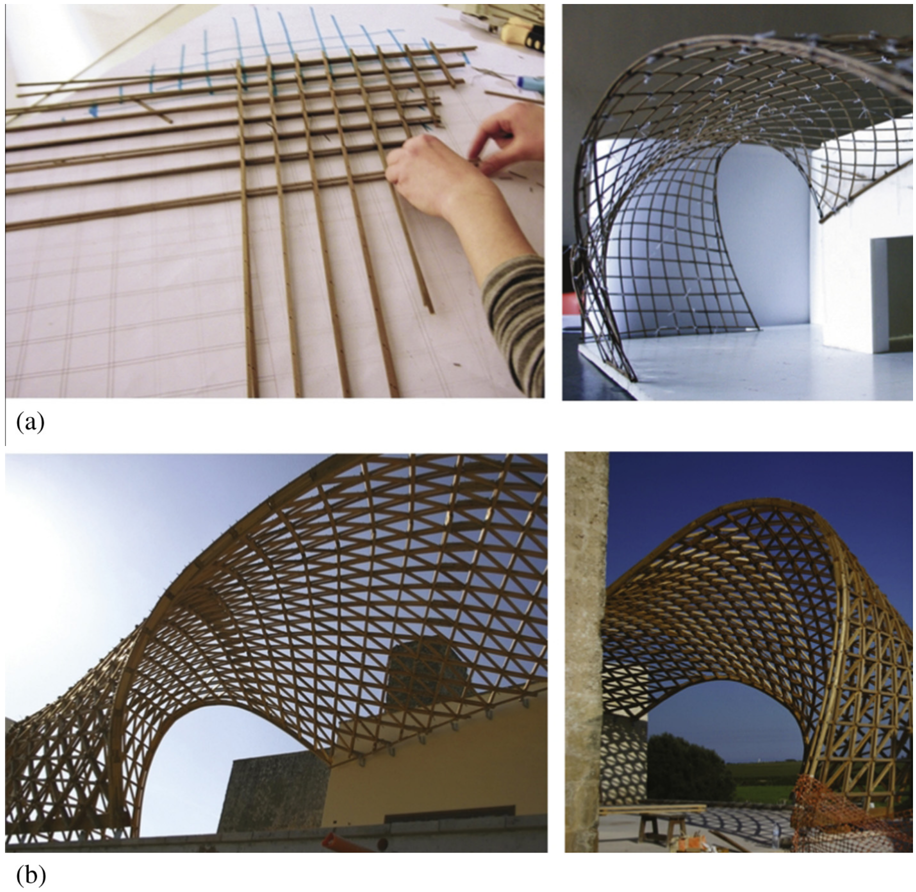

Fig. 1. Trio grid-shell, Lecce, Italy, 2010, by: CMMKM Architettura e Design: (a) Physical (scale) models were extensively used during the design phase and (b) realised structure.

and partial differential equations, implicit methods are preferred over explicit ones in describing the system's transient behaviour over the time domain (pseudo-time for static analyses). Implicit methods are generally preferred as they allow for larger (numerically stable) time increments to be considered and are insensitive to *numerical stiffness*. Emblematic in this regard is, for instance, the introduction to the Computer Graphic community of implicit integration methods for physically-based cloth simulation [33]. On the other hand, explicit methods have their own advantages, in particular: for those cases in which the given initial condition is 'very far' from the equilibrium solution, explicit formulations are more advantageous, since, the root-finding algorithm (e.g. the well known Newton–Raphson) allowing to 'implicitly' proceed over each time increment, works very well (quadratic convergence) when the integrating function is convex, whilst it is likely to fail otherwise.[1] This is a common situation when dealing with form finding analyses, in which, the problem's unknowns (namely, the structural shape) is sought by initializing the analysis with an arbitrary geometry, likely to experience gross deformations in converging to the equilibrium shape. This may explain the reason why, an explicit integration method such as the Dynamic Relaxation (DR) is a standard procedure in the form finding/analysis of tension structures [34].

Clarified that the choice of an explicit or implicit resolution method will mainly depend upon the problem to be solved, for what we are concerned in here regarding actively bent (and twisted) structural systems, the following considerations can be made:

- For a 'pure' simulation of the structure's physical behaviour, e.g. in order to simulate the construction (bending) process [35] or for instance, to assess the structure's behaviour under working loads, an implicit method will be more advantageous. In such a case, stiffness parameters will be physically meaningful, as well as the mass parameter (in case of dynamic analyses).
- On the other hand, for 'design-oriented' problems, e.g. form finding analyses, the geometrical shape (rather than stresses and deformations) is the main unknown in the problem. Accordingly, an explicit method will certainly be more tenacious in seeking a solution, and in such a case: masses, time-step size and stiffness parameters can have no physical meaning at all but will be (likely) set according to prescribed design parameters and/or numerical stability issues.

1.2. Discrete formulation: 3, 4 or 6 Degrees of Freedom?

According to continuum mechanics theory, a rod or beam is a three-dimensional object having one dimension (length) L much bigger than the other two. For instance, in case of rectangular cross-section, with b and h the cross-sectional width and height respectively:

$$L \gg b; \quad L \gg h \tag{1}$$

[1] In some cases such a 'limitation' inherent to implicit methods can have useful applications, as for instance, in the field of structural analysis, the critical buckling load of a structure can be obtained as the load increment at which the analysis fails to converge, since at that point the load–displacement curve becomes flat. Such a method was adopted, for instance, for the structural analysis of the Mannheim Multihalle grid-shell [11].