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# Analysis of three-dimensional curved beams using isogeometric approach

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ABSTRACT

There is an increasing trend among architects to utilize curved structural members as opposed to traditional straight structural members in the design of modern space structures. To handle the complex geometrical forms, new structural analysis methods are needed to accurately and efficiently analyze these structures. To address this issue, an isogeometric approach is presented in this paper for the formulation of finite elements for arbitrary spatially curved 3-D beams. These elements employ the same Non-Uniform Rational B-Spline (NURBS) interpolations as used in the CAD representations to interpolate both geometry and unknown fields in the finite element analysis, leading to a seamless transition between complex CAD models utilized by architects and the analysis models used by engineers. The effectiveness of the proposed method is demonstrated on a variety of curved structural geometries of varying complexities.



## 1. Introduction

Design of novel structural systems requires close cooperation between architects and engineers. The architects are primarily responsible for discovering structural forms while structural engineers are responsible for the analysis and design of those forms. In this iterative design process, computer aided design (CAD) software is used by architects to discover structural forms. Structural engineers, on the other hand, use finite element methods for analysis and design of those structural forms. This process requires CAD designs generated by the architects to be converted into analysis suitable models that can be used in the structural finite element analysis (FEA). This is a nontrivial process especially for the design of modern structural systems. There is an increasing trend among architects to turn away from traditional structural forms, utilizing straight beam–column members, and towards increasingly complex spatial designs, utilizing curved structural members. Moreover, when design changes are made both the CAD and the FEA models have to be updated. This is a difficult task, as geometries in CAD and FEA models are represented in fundamentally different ways. In CAD, geometries are usually represented by Non-Uniform Rational B-Splines (NURBS), while in traditional FEA geometries they are approximated by linear or quadratic Lagrangian interpolations [1,2]. Moreover, complex geometrical forms of spatially curved beam members involving non-uniform curvature and torsion cannot be *exactly* represented in traditional FEA using linear or quadratic interpolations. Furthermore, straight beam theories are used when approximations are made with linear elements, which may lead to additional FEA error. To address these issues, new analysis methods are needed that can: (a) exactly represent the CAD geometries of curved 3-D structural forms, and (b) accurately describe the mechanical response of curved structural members.

Isogeometric analysis (IGA) is introduced by Hughes et al. [3] as a way of bridging the gap between CAD and computer aided engineering (CAE) such as finite element analysis. IGA employs Non-uniform Rational B-Splines (NURBS) as the basis for interpolating both geometry and solution spaces. As NURBS are developed to represent curves and surfaces in CAD, IGA uses the same geometrical representation as a model derived from CAD, thus allowing structural analysis to be performed directly on the exact CAD geometry. An additional advantage of using IGA is that the order and continuity of interpolation basis can be increased efficiently and robustly through *k*-refinement [3,4]. This is a valuable property for interpolation of fields that require high-order continuity, and leads to results with higher accuracy [5,6]. Moreover, even if a solution space requires only $C^0$-continuity, a higher degree of continuity of interpolation functions may still lead to more accurate results because of the increased smoothness of solution spaces that provide significantly better numerical approximation. In the

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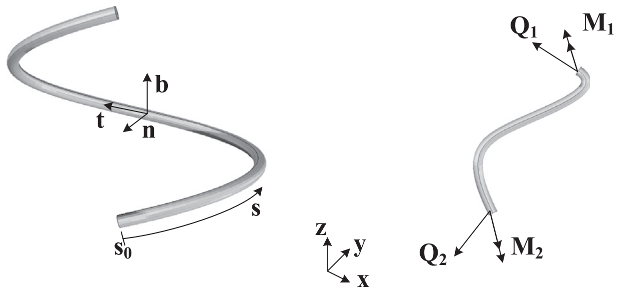



**Fig. 1.** Curved beam – Frenet–Serret basis and beam section with internal forces.

past, IGA has been successfully used in the analysis of static problems [3,4], wave propagation and fluid–structure interaction problems [4], structural optimization [7], free and forced vibration of linear and nonlinear structures [8,9] and so on. In this study, the use of isogeometric analysis is explored for curved 3-D beam structures.

The mechanics of 3-D curved beams has been studied for many decades and is well understood. Most common beam formulations are based on the differential geometry of curves in $\mathbb{R}^2$ or $\mathbb{R}^3$ and are developed in the so-called "natural" coordinate system. In general, the closed form stiffness matrices are not available for arbitrary curved beam elements; however, attempts have been made to derive closed form stiffness matrices for simple circular and parabolic 2-D geometries [10,11]. In some recent studies, direct numerical integration of the 3-D beams in the global system has been proposed, however, arbitrary 3-D spatially curved beams cannot be handled by this method [12]. In most cases, numerical methods based on finite element approximations are used to describe the behavior of arbitrary curved beams. The standard displacement based Lagrangian FEA formulations suffer from shear and membrane locking issues, however. To address these locking issues, mixed and hybrid finite elements for 2-D and 3-D curved beams has been proposed [13–16]. Nevertheless, the CAD geometries are still approximated in these methods. Some recent studies have investigated locking issues in IGA based 2-D curved beams with simple geometries [17,18]. As these studies were mostly focused on locking issues, only simple geometries for 2-D cases were employed and relevant implementation details for 3-D spatial curved beams are missing. Thus, to the best knowledge of the authors the use of Galerkin based IGA for arbitrary spatially curved 3-D beams has not been explored.

In this study, the isogeometric approach is used to formulate finite elements utilizing the theory of curved beams in space. Using IGA, the structural analysis is performed directly on the exact geometrical description of curved members given by the CAD models. Thus, an arbitrarily curved beam member that can be represented in CAD can be effectively analyzed using the proposed isogeometric approach. In addition to reducing modeling errors that come from geometrical and mechanical approximations, this eliminates the amount of time needed to be spent for updating the finite element models after geometrical changes are made in CAD designs. The locking issues in the proposed formulation are handled by using high order NURBS interpolations, and the effectiveness of the proposed method is demonstrated on a variety of curved beam geometries of varying complexities including geometries with non-uniform torsion and curvature. All the relevant implementation details for a general 3-D case that are needed for practical engineering applications are provided, and a potential implementation pitfall due to the discontinuity of the Frenet–Serret basis is pointed out and is appropriately addressed. The paper is organized as follows: Section 2 discusses the theory of curved beams in space. Section 3 gives background on IGA, while Section 4 presents the element formulation and implementation details. A locking case study and different examples of curved beams are presented in Section 5. Finally, the important conclusions of this paper are discussed in Section 6.

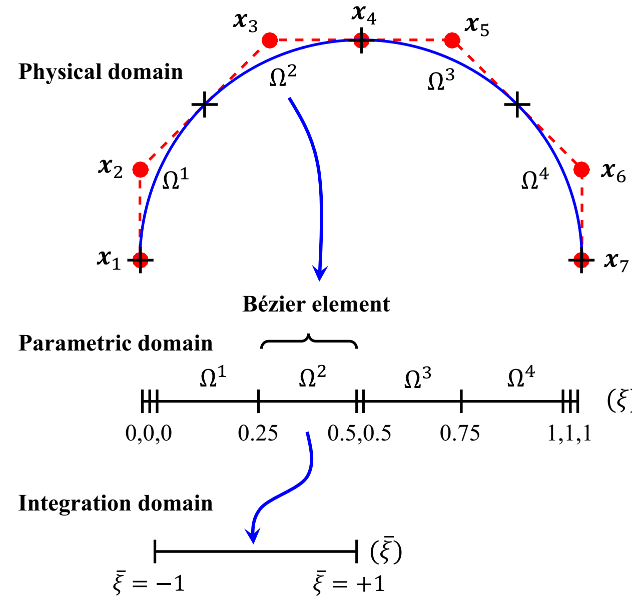
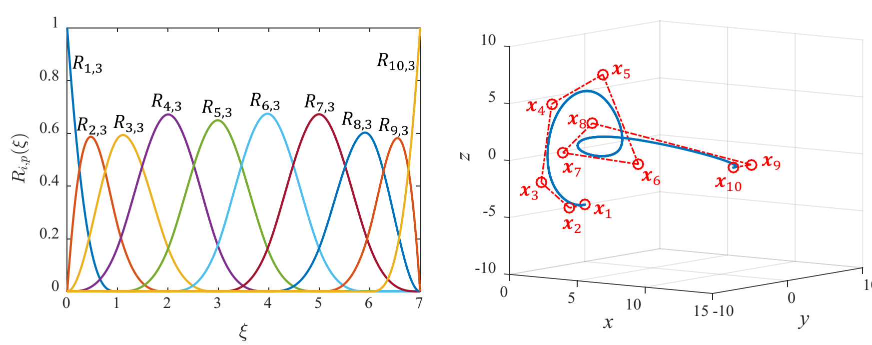



**Fig. 3.** IGA discretization of a curved beam.

## 2. Curved beam theory in 3-D

### 2.1. Frenet–Serret frame and geometry of curved beams

Consider an arbitrary regular curve $\boldsymbol{r}(s) : [0, L] \to \mathbb{R}^3$ embedded in $\mathbb{R}^3$ and parameterized by arc-length parameter $s$. For a 3-D



(a) NURBS basis functions (b) NURBS curve

**Fig. 2.** NURBS basis functions and NURBS curve with control points $\{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{10}\}$ based on an open knot vector $\Xi = \{0, 0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 7, 7, 7\}$.