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Stability and free vibration analyses of orthotropic 3d beam–columns with singly symmetric section including shear effects

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ABSTRACT

A complete beam–column classification and the corresponding characteristic equations for the stability and undamped natural frequencies of 3D orthotropic Timoshenko beam–columns with singly symmetric closed section and with elastic end connections subjected to an eccentric end axial load are presented and derived using three different approaches. The first two approaches are those by Engesser and Haringx that include the shear component of the applied axial force proportional to the slope (du/dx and dv/dx in planes xz and yz, respectively) and to the angle of rotation of the cross-section (θ_x and θ_y in planes yz and xz, respectively) along the span of the member, respectively. The third approach is a simplified formulation based on the classical Euler theory that includes the effects of shear deformations but neglects the induced shear component of the applied axial force along the member. The proposed methods and characteristic equations are capable of determining the critical axial loads and undamped natural frequencies of beam–columns with elastic end connections. Four comprehensive examples are included that show the effectiveness and simplicity of the proposed method and the results obtained are compared with experimental results available in the technical literature. It is shown that: (1) the natural frequencies and critical axial loads of beam–columns increase as the shear stiffness GA_s, the degrees of fixity and lateral bracings at the ends of the member increase; (2) the natural frequencies calculated using the three approaches are identical to each other when the applied axial load is zero; (3) the critical axial load in compression using the Engesser approach is lower than the one obtained using the Haringx approach; (4) the critical axial loads in compression are highly affected by the degree of flexural fixity at the supports, but those in tension are not affected much; and (5) the Haringx approach is the only one among the three approaches capable of capturing the phenomena of tension buckling observed in seismic isolators.

1. Introduction

The static and vibration analyses of beams and beam–columns are of great importance in structural dynamics, aerospace and earthquake engineering. The stability and dynamic behavior of beam–columns have been studied by numerous researchers and treated in the literature ([1–6], among others) using different methods. The combined effects of shear forces and bending deformations along the member on its critical axial load of prismatic beam–columns were first studied by Engesser in 1891 and later by Nussbaum in 1907. Timoshenko and Gere [7] presented these effects on the static lateral buckling of prismatic and built-up columns (laced, with batten plates, and with perforated cover plates) along with the historical contributions of Engesser, Nussbaum, Prandalt, Olsson, Haringx, and others. Additional details are given by Bazant and Celodin [8].

The vibration analysis of framed structures modeled with 2D beams and columns have been presented by Farghaly and Shebl [9] and Aristizabal-Ochoa [10–13] among many other researchers. On the other hand, the dynamic response of 3D beams and beam columns have been studied by Banerjee [14], Rafezy and Howson [15], and Viola et al. [16]. The free vibration of a 3D-orthotropic and uniform shear beam–column with generalized boundary conditions subjected to an eccentric end axial load in addition to a linearly distributed eccentric axial load along its span has been developed by Aristizabal-Ochoa [17]. Monsalve-Cano and Aristizabal-Ochoa [18] presented the characteristic equations for the undamped natural frequencies and buckling loads of an orthotropic singly symmetrical 3D Timoshenko beam–column with generalized support conditions using the Haringx approach only.

The effects of the shear force components induced by the applied axial force on FRP and sandwich columns and elastomeric bearings have been investigated experimentally and analytically by Kelly [19], Roberts [20], Aristizabal-Ochoa [21], Bai and Keller

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Nomenclature

A gross-sectional area of the beam–column
A_{sx} and A_{sy} effective shear areas along the x- and y-axes, respectively
A_1, A_2, A_3, A_4, A_5 and A_6 constants required in the vibration analysis of the beam–column in the yz-plane
E_z elastic modulus of the beam–column along the z-axis
F_1, F_2, F_3 and F_4 constants required in the vibration analysis of the beam–column in the xz-plane
G_x and G_y transverse shear moduli of the beam–column along the x- and y-axes, respectively
G_{xy} shear modulus of the beam–column under torsion
H_x and H_y shear force along the member in the x- and y-direction, respectively
I_x and I_y second moment of area of the member cross section about the x- and y-axes, respectively
I_α torsional inertia per unit of length of the beam–column about z-axis
J torsional moment of inertia of the cross section of the beam–column
J_{ax}, J_{ay} and J_{bx}, J_{by} rotational inertias of the masses at ends A and B about the x- and y-axes, respectively
$J_{a\psi}$ and $J_{b\psi}$ torsional inertias of the attached masses at ends A and B about the z-axes, respectively
κ_{ax}, κ_{bx} and κ_{bx}, κ_{by} stiffness of the elastic flexural connections at ends A and B about the x-and y-axes, respectively
$\kappa_{a\psi}$ and $\kappa_{b\psi}$ stiffness of the elastic torsional connections at ends A and B, respectively (force × distance/radian)
L span of the beam–column
M_a and M_b rigid masses attached at A and B ends of the beam–column, respectively
$M_x(\xi)$ and $M_y(\xi)$ bending moment along the beam–column about the x- and y-axes, respectively
$\bar{m}$ mass per unit length of the beam–column
P end axial load applied at the centroid of the cross section with coordinates $(x_\alpha,0)(+ \text{tensile})$
$\bar{m}L^2r_x^2$ and $\bar{m}L^2r_y^2$ rotatory inertias of the beam–column about the x- and y-axes, respectively
S_{ax}, S_{ay} and S_{bx}, S_{by} stiffness of the lateral bracings at ends A and B along the x- and y-axes, respectively
$u(z,t)$ lateral deflection of the shear center of the member along the x-axis
$v(z,t)$ lateral deflection of the shear center of the member along the y-axis
v_α lateral deflection of the centroidal line of the member along the y-axis
t time
T torsional moment
z centroidal axis of the beam–column
γ_x and γ_y shear distortion of the member cross section caused by transverse shear in the x- and y-directions, respectively
$\psi(z,t) = \Psi(z)\sin\omega t$ torsional rotation about the shear center S along the z-axis of the member
θ_x and θ_y bending rotations of the member cross section about the x- and y-axes

[22], Bazant [23], Fleck and Sridhar [24], Hoff and Mautner [25], Attard and Huntt [26] and Buckle, Nagarajaiah and Ferrell [27]. More recently, a complete column classification and corresponding stability equations that evaluate the elastic critical axial load of prismatic columns with sidesway uninhibited, partially inhibited, and totally uninhibited including the effects of bending and shear deformations and semi-rigid connections using the three approaches (Engesser, Haringx and Euler) have been proposed by Aristizabal-Ochoa [28,29].

The main objective of this paper is to present a complete beam–column classification and the corresponding characteristic equations for the undamped natural frequencies and modes of vibration of an orthotropic singly-symmetrical 3D Timoshenko beam–column with generalized end conditions (i.e., with semi-rigid bending restraints and lateral bracings as well as lumped masses at both ends) subject to a constant eccentric axial load at both ends using three different approaches. The first two approaches are those by Engesser and Haringx (fully explained in detail by Aristizabal-Ochoa [28], and Timoshenko and Gere [30]) that include the effects of the shear component of applied axial force proportional to the total slope and to the angle of rotation of the cross section along the span of the member, respectively. The third approach is a simplified formulation based on Euler theory that includes the effects of shear deformations but neglecting the shear component of the applied axial force along the span of the member.

2. Structural dynamic model

The proposed model is an extension of those presented previously by the authors [18,28]. Consider the Timoshenko beam–column AB of length span L with singly symmetrical cross section as shown in of Fig. 1. It is assumed that the member AB to be: (1) prismatic with perfectly straight centroidal axis z; (2) subject to a end axial load P applied along the z-axis; (3) with two rigid masses attached at A and B of magnitudes M_a and M_b with rotatory inertias $J_{ax}, J_{ay}, J_{a\psi}$ and $J_{bx}, J_{by}, J_{b\psi}$ about the x, y and z axes, respectively; (4) elastic torsional connections with stiffnesses $\kappa_{a\psi}$ and $\kappa_{b\psi}$, elastic bending connections with stiffnesses κ_{ax}, κ_{bx} and κ_{ay}, κ_{by} about the local principal x- and y-axes, and end lateral elastic connections with stiffnesses S_{ax}, S_{bx} and S_{ay}, S_{by} along the local principal x- and y-axes at ends A and B, respectively. The end connections are related to all six possible degrees of freedom (bending, torsional, axial and transverse directions) at each extreme of the beam–column. In the technical literature, these are generally assumed as perfectly rigid (infinity stiffness) or perfectly hinged (zero stiffness) connections in the design of beam-to-beam, beam-to-column, and column-foundation connections.

The properties of member includes: mass per unit length $\bar{m}$, moments of inertia I_x and I_y about its cross section main centroidal axes x and y; torsional moment of inertia J and torsional shear modulus G_{xy}; cross-section area A and axial modulus E_z; effective shear-areas A_{sx} and A_{sy} with the corresponding shear moduli G_x and G_y. Also notice that letters E, H and S are used throughout this manuscript for differentiate between Engesser, Haringx, and Simplified Euler approaches, respectively.

3. Governing equations in the *xz*-plane

The governing differential equations of a Timoshenko beam–column for buckling and free vibration in the xz plane utilizing the three different approaches are as follow:

$$G_xA_{sx}\left(u'' - \theta_y'\right) + Pu'' - \bar{m}\ddot{u} = 0 \tag{1E}$$

$$E_zI_y\theta_y'' + G_xA_{sx}(u' - \theta_y) - \bar{m}L^2r_y^2\ddot{\theta}_y = 0 \tag{2E}$$