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Crack width analysis of reinforced concrete members under flexure by finite element method and crack queuing algorithm

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ABSTRACT

In reinforced concrete design, it is necessary to evaluate the crack widths so as to ensure compliance with the design codes. However, crack width analysis is not easy and so far only empirical formulas, which do not agree with each other, are available for rough estimation. Particularly, the smeared crack models, which do not allow for bond–slip of reinforcing bars, would not give any crack widths. On the other hand, the discrete crack models are difficult to apply because of the need to adaptively generate discrete crack elements to follow the crack formation. Herein, a new finite element method for discrete crack analysis, which does not require the use of discrete crack elements, is developed. The reinforcing bars are modeled by discrete bar elements and their bond–slip is allowed for using interface elements. Moreover, a crack queuing algorithm is employed to simulate the stress redistribution during cracking and a cracking criterion based on both tensile strength and fracture toughness is adopted to cater for stress concentration at crack tips for correct prediction of crack number, spacing and widths.

1. Introduction

In the design of reinforced concrete structures, we need to consider not only the strength and ductility under ultimate load but also the serviceability and durability under servicing load. One major factor to be considered in the serviceability and durability design is the crack width of the concrete, which has to be limited to a certain maximum allowable value to avoid aesthetic, water leakage and steel corrosion problems. However, there has been little research on this important topic. Up to now, only empirical formulas for crack width prediction, which do not agree with each other, are given in design codes, such as BS 8110-2: 1985 [1], ACI 224R-01 [2] and Eurocode 2: 2004 [3]. One probable reason is that the phenomena of crack initiation, propagation and widening are fairly complicated and difficult to analyze. This is because at the crack tips, there are stress concentrations and as the cracks widen, bond-slip of the steel reinforcing bars takes place. Even the finite element methods, which do not allow for the fracture toughness of concrete, bond-slip of reinforcing bars and stress redistribution during crack formation, are not capable of analyzing the crack number, spacing and widths in the concrete.

Finite element methods for crack analysis of reinforced concrete structures may be categorized into the discrete crack model and the smeared crack model. The discrete crack model was first proposed by Ngo and Scordelis [4] in the 1960s. In their model, a discrete crack element is inserted into the concrete to simulate the formation of a crack during loading. After inserting the discrete crack element, the surrounding concrete is separated by cutting the concrete elements at the crack and assigning two different nodes to the concrete at the same point, one for the concrete at one side of the crack and the other for the concrete at the other side. However, the crack locations have to be pre-determined so that the discrete crack elements can be inserted right at the beginning of analysis. For cases in which the crack locations are not known beforehand, this model does not really work. On the other hand, Nilson [5] developed a model in which the cracks can propagate along the boundaries between the concrete elements. However, forcing the cracks to propagate only along the element boundaries could result in a rather awkward crack pattern when the element size is large and might restrain the propagation of cracks when there are multiple distributed cracks [6]. For this reason, the discrete crack elements should be adaptively inserted according to the actual stress field obtained during the analysis [7–9]. But this requires element re-meshing and node re-numbering in each iteration step. As a result, the computer power and time required are generally very high, making such adaptive discrete crack model rather difficult to apply.

Due to its simplicity and easier application, the smeared crack model is more commonly used. Rashid [10] was among the earliest researchers who developed the smeared crack model in the 1960s.

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In the smeared crack model, the formation of a crack is simulated by changing the constitutive properties of the whole concrete element containing the crack. Basically, after the formation of a crack, the concrete is assumed to have very little stiffness and strength in the direction perpendicular to the crack. Since the constitutive properties of the whole concrete element are changed, the crack is in effect smeared within the volume of the concrete element. However, in most smeared crack models, such as those of Cope et al. [11] and Gupta and Akbar [12], the steel reinforcing bars are also smeared within the volume of the concrete element by adding their constitutive stiffness matrix to the concrete element. As a result, the bond–slip of the steel reinforcing bars, which has great effect on the crack width, is ignored. The smeared crack model may be further categorized into the non-rotating crack model [10,13,14] and rotating crack model [15,16]. In the non-rotating crack model, the crack directions are assumed to be fixed once the cracks are formed but in the rotating crack model, the cracks are allowed to rotate with the principal strain directions. Anyway, although the existing smeared crack models do allow us to take into account the effect of crack formation on the overall stiffness and strength of the reinforced concrete member being analyzed, they are not capable of predicting the crack number, spacing and widths in the concrete.

There are also finite element methods, which employ the smeared crack model for discrete crack analysis. This strategy would avoid the need to adaptively generate discrete crack elements to simulate crack formation and propagation, but to accurately follow the crack pattern, the concrete element mesh has to be very fine. Using such kind of method, Riggs and Powell [17] have developed a model, which is capable of accounting for the interfacial shear dilation and transfer across the cracks. However, as the bond–slip of the steel reinforcing bars has not been accounted for, this model is also not capable of predicting the crack number, spacing and widths.

The authors are of the view that to predict the crack number, spacing and widths, even accounting for the bond–slip of the steel reinforcing bars is not enough. Since there would be stress concentration at the crack tips, the fracture toughness of the concrete needs to be taken into account. Moreover, during crack formation and propagation, there would be immediate stress redistribution in the vicinity of the cracks, which would relieve the tensile stresses perpendicular to the cracks to avoid the formation of other cracks close to the newly formed cracks. In conventional finite element methods, such stress relief is not taken into account and consequently, many closely spaced cracks are often formed in the same iteration step. The second author has previously developed a model [18], in which the interfacial transition zones at aggregate–cement paste interfaces are modeled by discrete crack elements, the stress concentration and fracture toughness are taken into account in the cracking criteria, and stress relief during crack formation is simulated using a crack queuing algorithm of allowing only one crack to form at a time and evaluating the stress redistribution so caused by reanalyzing the concrete stresses before allowing another crack to form. This discrete crack model has the same problem with the other discrete crack models in being quite difficult to apply. Herein, the discrete crack model is changed to a smeared crack model for easier application. To enable such change, the bond–slip of steel reinforcing bars, fracture toughness of concrete and stress redistribution during crack formation are accounted for using special numerical techniques.

2. Finite element method

In the finite element analysis, the concrete is modeled by 2-dimensional plane stress 3-noded triangular elements, the steel reinforcing bars are modeled by 1-dimensional 2-noded bar elements, and the steel bar-concrete interfacial bond is modeled by 1-dimensional 4-noded interface elements. In order to simulate the post-crack and post-peak behavior of reinforced concrete structures, secant stiffness is used in the finite element formulation [19].

2.1. Concrete elements

The concrete is modeled by a 3-noded triangular element, which is allowed to have tensile cracks formed inside the element. In each concrete element, the axial and shear strains in the global coordinates are first evaluated in terms of the nodal displacements, and then the axial and shear strains in the local coordinates (taken to be the principal strain directions before cracking and fixed to be the crack directions after cracking) are transformed from those in the global coordinates. To allow for the nonlinear behavior, the biaxial stress–strain relations in the local coordinates are simplified into two uniaxial stress–strain relations such that the axial stress in each coordinate axis direction is taken only as a function of the equivalent uniaxial strain in that coordinate axis direction, as proposed by Ng et al. [20]. The equivalent uniaxial strains, denoted by ε_1^e and ε_2^e, are defined by the following equations:

$$\varepsilon_1^e = \frac{1}{1 - v_1 v_2}(\varepsilon_1 + v_2 \varepsilon_2) \tag{1}$$

$$\varepsilon_2^e = \frac{1}{1 - v_1 v_2}(\varepsilon_2 + v_1 \varepsilon_1) \tag{2}$$

where ε_1 and ε_2 are the axial strains in the local coordinates, and v_1 and v_2 are the Poisson's ratios. The sign conventions for the axial strains and equivalent uniaxial strains are tension positive and compressive negative.

When the equivalent uniaxial strain is compressive, the stress–strain curve developed by Desayi and Krishnan [21], which has clear ascending and descending branches and a peak at secant modulus equal to half the initial elastic modulus, is used. Mathematically, the compressive stress–strain curve is given by:

$$\sigma = \frac{E_0 \varepsilon}{1 + (\varepsilon / \varepsilon_{c0})^2} \tag{3}$$

in which σ is the axial stress, ε is the equivalent uniaxial strain, E_0 is the initial elastic modulus and ε_{c0} is the axial strain at peak, as illustrated in Fig. 1. Note that ε_{c0} is related to the axial stress at peak σ_{c0} by $\varepsilon_{c0} = 2\sigma_{c0}/E_0$.

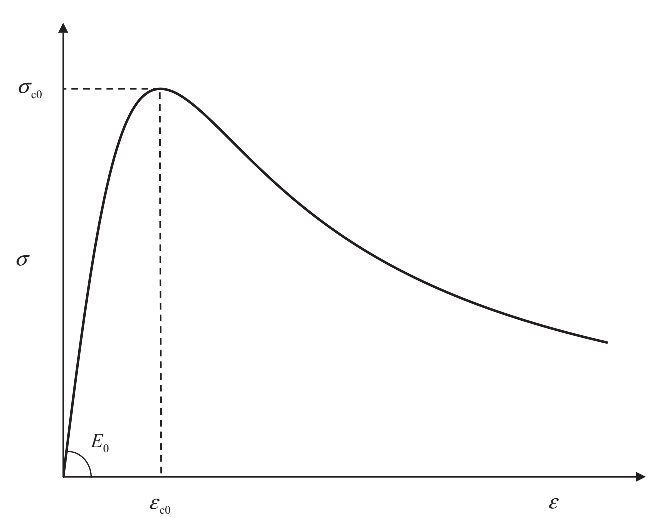

Fig. 1. Uniaxial compressive stress–strain curve of concrete.