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Short communication

On the use of fixed point theory to design coupled core walls

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ABSTRACT

The use of Fixed Point Theory (FPT) to optimize the design of coupling beams in coupled core wall (CCW) systems is demonstrated. The basis for optimization is minimizing the transmissibility of horizontal ground motion by appropriately linking two coupled wall piers having different dynamic properties with beams having appropriate stiffness and damping characteristics. Using 21 example CCW structures illustrating a range of pier properties, it was shown that the resulting optimization of coupling stiffness is quite small and other design considerations will require stiffer, non-optimal coupling beams. Nonetheless, the potential to leverage the small amount of coupling available in a 'slab-coupled' series of wall piers in order to reduce transmissibility is suggested by the findings of this study.

1. Introduction

Hull and Harries [1] identified Fixed Point Theory (FPT) as having potential applications to the performance-based design (PBD) of coupled core wall (CCW) systems. They identified the potential transition from CCW behavior under service lateral loads to a system of linked wall piers (LWP) under design seismic loads. Their work focused on the performance of the LWP system. Hull and Harries proposed a novel measure of performance: minimization of transmissibility of horizontal ground motion through the optimization of coupling beam stiffness resulting in the optimal engagement of two wall piers. Transmissibility is simply defined as the ratio of structural deflection to input horizontal ground motion. With the exception of very stiff structures, transmissibility is typically greater than unity. In a structure composed of multiple linked structural elements, transmissibility is affected by the ratio of dynamic properties of the coupled elements and the connection between these. By varying the relationship between dynamic properties of elements, transmissibility may be changed. Structures composed of dynamically identical components cannot be optimized using FPT; in such a case transmissibility is only a function of the sum of the element stiffnesses [1].

In this paper, the practical application of optimizing coupling beam stiffness between dynamically dissimilar wall piers using FPT will be investigated. The hypothesis being that the stiffness of the coupling beams for a given set of wall piers may be optimized to improve the CCW and subsequent LWP response to earthquake excitation. As shown in Fig. 1, each wall pier is idealized as a single degree of freedom (SDOF) system having mass, stiffness and damping, m_i, k_i and c_i. The stiffness and damping (k_b and c_b, respectively) of the coupling continuum are represented by a spring and dashpot system and may be optimized so as to minimize lateral deflections X_1 and X_2 for a ground excitation U [2].

In this study, CCW prototype structures similar to those previously identified by Harries et al. [3] are used. These are 12-storey structures that have seven individual pier geometries labeled A through G, shown schematically in Table 1. The thickness of the wall piers is 0.35 m and the uniform storey height is 3.6 m. The other dimensions and resulting wall pier areas and moments of inertia are presented in Table 1. The coupling beam geometric information is not relevant at this point; indeed, this analysis is intended to lead to coupling beam stiffness requirements. The individual wall piers are paired into two-pier CCW systems, each pier is matched with each other pier resulting in 28 unique analysis cases. Optimal coupling of identical wall piers based on transmissibility is meaningless (i.e.: Wall A coupled to Wall A); thus the number of unique analyses is 21. For example, case 16 (Wall D coupled to Wall E) is shown in Fig. 2.

2. Derivation of the equivalent SDOF structure

In order to model each MDOF wall pier as a SDOF system, it is represented by a massless beam–column member supporting a

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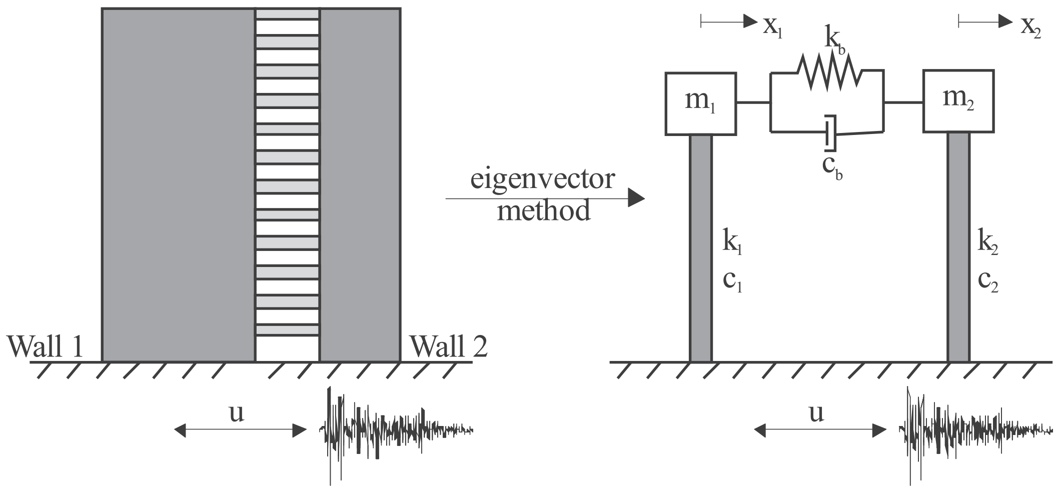
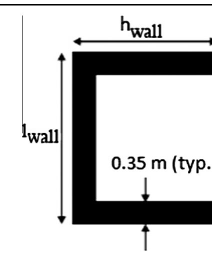


Fig. 1. Idealized 2DOF system for application of fixed point theory (adapted from [1]).

Table 1
Wall pier dimensions used in FPT analysis [3].

Wall	Wall flange (h_{wall}) m	Wall web (l_{wall}) m	Gross wall area (A_g) m^2	Gross wall inertia (I_g) m^4	Wall geometry
A	7.00	9.00	7.80	40.20	
B	6.00	3.00	5.01	18.00	
C	4.00	3.00	3.60	5.83	
D	5.00	6.00	5.35	13.86	
E	3.00	6.00	3.96	3.32	
F	3.00	3.00	2.91	2.61	
G	4.00	9.00	5.70	8.51	

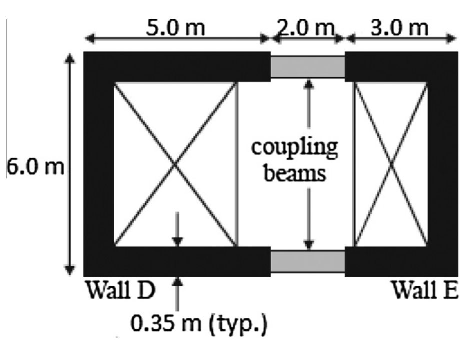


Fig. 2. Example of prototype CCW Plan: Case 16: coupled Walls D and E.

lumped mass at the top (Fig. 1). Each beam–column is assigned geometric and material properties of the wall pier. The eigenvector method [4] is used to establish the equivalent SDOF mass. For each analysis case, the mass of the MDOF wall pier takes the form of a diagonal mass matrix, $\mathbf{M}_i$, with the diagonal values representing the portion of the storey mass assigned to each wall pier, i, based on its relative sectional area.

Each MDOF cantilever wall pier is assumed to have a fixed base and a single DOF at each floor level. The resulting stiffness matrix for each wall is therefore:

$$K_i = \begin{bmatrix} 2k_{iX} & -k_{iX} & 0 & \dots & \\ -k_{iX} & 2k_{iX} & -k_{iX} & 0 & \\ 0 & -k_{iX} & 2k_{iX} & & \\ \dots & 0 & & 2k_{iX} & -k_{iX} \\ & & & -k_{iX} & k_{iX} \end{bmatrix} \quad (1)$$

In which the lateral stiffness associated with each floor, X, of each wall, i, is $k_{iX} = 12EI_{iX}/h^3$.

The eigenvalues, ω_{in}, representing the natural frequencies, and the eigenvectors, φ_{in}, representing the solution to the undamped free vibration equation of each wall, $\mathbf{M}_i\ddot{X} + \mathbf{K}_iX = 0$, are calculated. The effective equivalent SDOF modal mass of each wall, M_{in}, corresponding to each mode, n, is [5]:

$$m_{in} = \frac{\left(\sum_i^N m_i\varphi_{in}\right)^2}{\varphi_{in}^T M_i \varphi_{in}} \quad (2)$$

where N is the number of degrees of freedom (storeys) in the MDOF structures, and m_i is the storey mass associated with each DOF. The equivalent SDOF stiffness of each wall, K_{in}, is defined as [5]:

$$k_{in} = \omega_n^2 m_{in} \quad (3)$$

For the present study, only the fundamental natural frequency is considered; thus $n = 1$ in all equations. Due to the assumed vertical uniformity of the wall piers, considering only the first mode results in a modal participation factor equal to greater than 0.90 in all cases [6].

3. Fixed point theory

Using the SDOF systems derived in the previous section, FPT is used to determine optimal values of coupling stiffness, k_b, and damping, c_b, that result in the lowest transmissibility for the 2DOF system shown in Fig. 1. The transmissibility is defined as the ratio of the structure top displacement (x_i) to the displacement induced by the ground motion (u). The complete derivation of the