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The effective length of columns in multi-storey frames

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ABSTRACT

Codes of practice rely on the effective length method to assess the stability of multi-storey frames. The effective length method involves isolating a critical column within a frame and evaluating the rotational and translational stiffness of its end restraints, so that the critical buckling load may be obtained.

The non-contradictory complementary information (NCCI) document SN008a (Oppe et al., 2005) to BS EN 1993-1 (BSI, 2005) provides erroneous results in certain situations because it omits the contribution made to the rotational stiffness of the end restraints by columns above and below, and to the translational stiffness of end restraints by other columns in the same storey.

Two improvements to the method are proposed in this paper. First, the axial load in adjoining columns is incorporated into the calculation of the effective length. Second, a modification to the effective length ratio is proposed that allows the buckling load of adjacent columns to be considered. The improvements are shown to be effective and consistently provide results within 2% of that computed by structural analysis software, as opposed to the up to 80% discrepancies seen using the NCCI (Oppe et al., 2005).

1. Introduction

Many codes of practice rely on the effective length method to assess the stability of frames. The effective length method allows the buckling capacity of a member in a structural system to be calculated by considering an equivalent pin ended column in Euler buckling. This paper will focus on the non-contradictory complementary information (NCCI) document SN008a [1] to BS EN 1993-1 [2], although many of the findings presented in this paper are also relevant to many other national codes of practice. The NCCI provides a simple method to determine the effective lengths of columns in multi-storey steel frames. Errors in this approach have been identified that arise as the method fails to correctly recognise the contribution made:

1. by adjoining columns, to the rotational stiffness of end restraints; and
2. by other columns in the same storey, to the translational stiffness of end restraints.

Issue (1) concerns both braced and unbraced frames. Using the NCCI [1] it is found that the stiffer an adjoining column, the greater the effective length of the column being analysed, which is counter-intuitive. This is demonstrated by considering columns AB and CD in Fig. 1. If AB and CD are stiffened and the loading unchanged, then the rotations at B and C are reduced. The deflected shape shows that the effective length of *BC* is reduced in this situation, whereas the equations of the NCCI [1] show it to increase, as shown later. A simple improvement to the method is proposed to address this, which incorporates the adjoining columns' axial load into the calculation of the effective length. The improvement is shown to be very effective and consistently provides results within 2% of that computed by structural analysis software.

Issue (2) concerns unbraced frames, and occurs because of the simplifying assumption made in the NCCI [1] that all columns in a storey buckle simultaneously and therefore columns in this storey have end restraints with zero translational stiffness. If the method is applied to unbraced frames where columns of varying stiffness exist in the same storey or columns have different applied loads, then significant errors will be encountered that are potentially unconservative, as seen in Section 3.2.1 below. To address this issue, a modification factor is adopted which is applied to the effective length ratio obtained using the sway design chart, and accounts for columns that will have end restraints with translational stiffnesses between zero (sway case) and infinity (non-sway case) and even negative translational stiffnesses. These are often called partial sway frames. The results obtained from using this factor are shown to be reliable and accurate.

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Nomenclature

C	carry-over factor
E	Young's modulus
η_x	distribution coefficient at node X
I	second moment of area
I_{IJ}	second moment of area of column IJ
κ	curvature of an elastic member
K_{IJ}	nominal rotational stiffness of column IJ
K_{XY}	nominal rotational stiffness of an adjoining column XY
K'_{IJ}	rotational stiffness of column IJ modified for axial load
K''_{IJ}	rotational stiffness of column IJ at node I modified for axial load and support conditions at node J
K''_{JI}	rotational stiffness of column IJ at node J modified for axial load and support conditions at node I
K''_{XY}	rotational stiffness of an adjoining column XY modified for axial load and far end support conditions
$\sum K_{c,I}$	sum of rotational stiffness of the columns converging at node I
$\sum K_{b,I}$	sum of rotational stiffness of the beams converging at node I modified for axial load and far end support conditions
$L_{E,IJ}$	effective length of column IJ
L_{IJ}	physical length of column IJ
M_x	moment at node X
P	applied compression
$P_{c,IJ}$	critical buckling load for column IJ given by Eq. (3)
$P_{E,IJ}$	Euler buckling load for column IJ given when $L_{E,IJ} = L_{IJ}$ in Eq. (3)
P_{IJ}	applied compression on column IJ
θ_x	rotation at node X
S	stiffness coefficient
v	deflection

1.1. Elastic stability

Buckling is an instability phenomenon in structural systems subjected to compression loads. In columns it is associated with the transition from a straight configuration to a laterally deformed state [3]. The critical load describes the load at which this transition occurs.

Critical loads can be calculated by solving for equilibrium of the laterally deformed column. Assuming deflections and rotations are small, the curvature of a member, κ, can be defined by Eq. (1). If the member is perfectly elastic and the material obeys Hooke's Law, deflection theory [4] states that the bending moment is proportional to the curvature, with the member's flexural stiffness as the constant of proportionality, Eq. (2):

$$\kappa = \frac{d^2 v}{dx^2} \tag{1}$$

$$M = -EI\frac{d^2 v}{dx^2} \tag{2}$$

where v is the deflection; E is Young's modulus, I is the second moment of area.

With the substitution $k^2 = P/EI$, the solution for the critical buckling load is given by Eq. (3) where the boundary conditions of the column are used to define the effective length:

$$P_c = \frac{\pi^2 EI}{L_E^2}, \tag{3}$$

where E is the Young's modulus, I is the second moment of area, P_c is the critical buckling load, and L_E is the effective length of the column.

1.2. Effective length

The effective length, L_E, depends on the boundary conditions of the column as shown for example in [5]. A pin ended elastic column will have a buckled configuration of a sinusoidal wave. The distance between points of contraflexure, which defines the effective length, is critical in evaluating the stability of the column. Effective lengths given in the codes are generally greater than the theoretical values, as full rigidity at supports is difficult, if not impossible, to achieve.

Theoretical analysis uses idealised end restraints, whose translational and rotational stiffnesses are set to either zero or infinity. In some instances it may be acceptable for the designer to assume a column has these idealised end restraint conditions, especially for preliminary design purposes when a more rigorous analysis is to follow, but care is needed due to the substantial influence that end restraints have on the buckling capacity. In most real structures, the rotational and translational stiffness of the end restraints is somewhere between rigid and free.

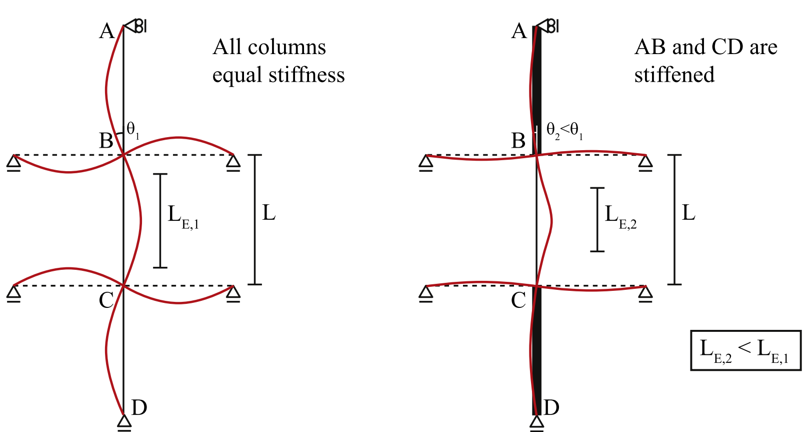

Fig. 1. The contribution of adjoining columns to rotational stiffness at end restraints.