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Bayesian model updating of a coupled-slab system using field test data utilizing an enhanced Markov chain Monte Carlo simulation algorithm

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ABSTRACT

Markov chain Monte Carlo (MCMC) simulation is applied for model updating of the coupled-slab system of a building structure based on field test data following the Bayesian theory. It is found that the identifiability of the model updating problem depends very much on the complexity of the class of models. By MCMC, the same algorithm can be used no matter the model updating problem is locally identifiable or not. The posterior joint probability density function (PDF) of model parameters is derived with consideration of the uncertainties from both the measurement noise and modeling error. To obtain a posterior PDF that is not analytically available in the complicated parameter space, an MCMC algorithm is proposed to sample a set of models in high-probability regions for the representation (or approximation) of the posterior PDF. The sampling process is divided into multiple levels, and individual bridge PDFs are constructed at each level that finally converged to the target posterior PDF. The samples move smoothly through each level and finally arrive at the important region of the target posterior PDF. A novel stopping criterion for the MCMC algorithm is proposed from the insight of the derivation of the posterior PDF. In the field test verification, the posterior marginal PDFs conditional on two model classes are obtained by the proposed MCMC algorithm, which provide valuable information about the identifiability of different model parameters.

1. Introduction

In structural engineering, great efforts have been made to develop sophisticated computer models of structures not only for the prediction of responses but also to provide insight into structural behavior for improvement of the structural design under various configurations. For these purposes, the structural model must be highly accurate to represent the behavior of the target structure. No matter how sophisticated the computer model is, modeling errors do exist. They come from various sources and can be categorized into three types. The first consists of mathematical modeling errors caused by the simplification of complicated real structural behaviors. These errors include simplifications in boundary conditions, connections, and geometric shapes. The second type consists of model order errors caused by the discretization of the continuous system to form a finite-element model (FEM). The third type consists of physical parameter errors caused by errors in the assignment of values to various model parameters (e.g., the Young's modulus of concrete that may not necessarily be a constant throughout the entire structure and the spring constant for capturing the semi-rigid behavior of structural joints). Structural model updating is crucial for obtaining accurate models when the measured structural responses are available. In addition, structural model updating is widely applied for damage detection [1–6] and structural health monitoring [7–11]. The methods for model updating can be categorized into deterministic and probabilistic methods.

The development of deterministic model updating methods is relatively mature [12]. Ahmadian et al. [13] investigated regularization methods for model updating, including those based on singular value decomposition, cross-validation, and L-curves. Jaishi and Ren [14] treated model updating as a constrained optimization problem, which minimized the discrepancy between the measured and model-predicted responses. In Jaishi and Ren's work [14], the objective function was formulated by considering frequency residual, mode shape-related function, and modal flexibility residual. Their method was verified using the ambient vibration data of a bridge. Other examples of deterministic methods can be found in Refs. [15–17]. Although deterministic methods have been successfully applied, there are some limitations. Most deterministic methods try to pinpoint a single solution and ignore other

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possible solutions resulting from the incomplete nature of measurement and the problem of modeling error. Moreover, the uncertainties associated with the results of model updating are not explicitly considered in deterministic methods.

Model updating is usually an ill-conditioned inverse problem in practice. For instance, the measured modal data are incomplete, implying that the full set of degrees-of-freedom (DOFs) presented in the computer model cannot be completely measured and that only a small portion of the modes can be identified. As a result, the solution may be nonunique or even unidentifiable. Modeling and measurement errors always induce a different order of uncertainties to a model updating result. Bayesian model updating methods provide a rational way to handle the aforementioned problems by incorporating a probability model. When the situation is globally or locally identifiable [18], the posterior probability density functions (PDFs) of the uncertain model parameters could be represented by the weighted sum of multiple Gaussian PDFs centered at finite isolated points. The marginal PDFs of the uncertain model parameters can be obtained by Gaussian approximation [19]. These isolated points are the "optimal models" in deterministic methods. Identifiability issues were later addressed in a unified manner [20]. It was found that for locally identifiable cases, equally important models would give different responses at the unobserved DOFs, indicating that the use of only one of these models was unreliable. An algorithm was developed to efficiently search the high-dimension parameter space using a network of trajectories to find all output-equivalent optimal models.

The Bayesian method in [18] used Gaussian distribution for approximation of the posterior PDF. When the levels of measurement noise and modeling error are high, approximation by Gaussian distributions is not necessarily accurate. To overcome this difficulty, Katafygiotis et al. [21] generated a series of well-structured points in the important region of parameter space in which the value of the posterior PDF was higher than a predefined threshold value (i.e., the subset of models with relatively high posterior PDF value). One advantage of this method is that the points generated in the important region are approximately equally spaced. Once this set of points is available, the approximation of the posterior PDF is straightforward and very accurate. However, the amount of computational effort grows as the number of uncertain model parameters increases. The practical application of this method is prohibited by its computational efficiency when the target structure is complicated with many uncertain parameters.

In this article, the finite element model of a coupled-slab system was updated using modal parameters identified from an ambient test [22,23]. For the real applications of model updating, the effects of both modeling error and measurement noise are relatively large when compared to numerical example or experimental case study under laboratory conditions, and the posterior uncertainties are usually high. For a given set of measurement, the model updating problem may change from identifiable to unidentifiable if the complexity of the model class is increased. The MCMC algorithm [24,25] provides a convenient way to calculate the posterior PDF of uncertain parameters no matter the problem is identifiable or unidentifiable. There are applications of MCMC for model updating. In Ref. [26], MCMC is applied to compare two stochastic model updating methods, namely covariance and interval model updating. The comparison is based on the measured modal parameters of a simplified aircraft model. In Ref. [27], the shadow hybrid Monte Carlo (SHMC) is proposed to overcome the limitation of hybrid Monte Carlo (HMC). The performance of SHMC for model updating is verified on an unsymmetrical H-shaped structure and a simplified aircraft by using the measured natural frequencies. The transitional Markov chain Monte Carlo (TMCMC) algorithm [25] is applied in [28] to investigate the uncertainties in Bayesian model updating. The framework proposed in [28] can reasonably predict the posterior uncertainties in model updating. The parallelized TMCMC is proposed in [29] to increase the computational speed. Two aerospace structures, the antenna reflector and satellite, are used to verify the proposed model updating method. The neural network is applied to reduce the computational time of the full finite element analysis. The verification results are encouraging. When compared to the TMCMC method, the proposed method adopted a different way to control the samples in each sampling level to approach the important regions of the posterior PDF. Furthermore, the proposed formulation of the target PDF in the Metropolis–Hastings (MH) algorithm is also different from that in the TMCMC method. It must be pointed out that most existing research work on the use of MCMC in structural model updating or structural health monitoring, the target structures are usually numerical examples [30] or simple experimental case studies under laboratory conditions [31]. In this study, a coupled-slab system of a building structure is considered, and the model updating is based on a set of measured modal parameters, which were obtained through an ambient test under the normal operation of the building. This represents a real application example of MCMC based Bayesian model updating. Considering that structural health monitoring of civil engineering structures has been widely studied [32–34], this important step is certainly helpful for the application of structural model updating in long-term structural health monitoring of civil engineering structures, such as buildings.

In this study, a Markov chain Monte Carlo (MCMC) algorithm is proposed for Bayesian model updating, which is conducted by sampling from the posterior PDF. The sampling process is divided into multiple levels to explore the parameter space efficiently. This multilevel idea is similar to simulated annealing and was applied by Beck and Au [24] for Bayesian model updating of a two-story shear building model based on simulated data. A novel stopping criterion is developed in this study to improve the method in [24]. Kernel density estimation [35–37] is performed to efficiently obtain the posterior marginal PDFs for prediction of the posterior uncertainties. The practical value of the proposed method is demonstrated by its application in a coupled-slab system based on field test data. In the field test verification, the enhanced MCMC algorithm is applied two times for approximation of the posterior marginal PDFs of uncertain parameters conditional on two model classes. The case study showed that valuable information about the identifiability of the model updating problem and the corresponding model parameters could be obtained by the approximated posterior marginal PDFs.

2. The enhanced MCMC algorithm for Bayesian model updating

In the proposed algorithm, a posterior PDF that considers the effects of both measurement noise and modeling error is first derived. A multilevel sampling scheme is next introduced to generate samples for the approximation of the target posterior PDFs. A novel stopping criterion is proposed to allow accurate estimation of the posterior uncertainty. Finally, the procedures of the proposed algorithm are summarized for practical applications.

2.1. Formulation of posterior PDF

The formulation of the posterior PDF of the set of model parameters to be identified is needed before samples can be generated by the MCMC algorithm. By following the Bayesian theorem, the posterior PDF of the uncertain model parameter vector $\mathbf{x}$ conditional on the measured modal data $\mathbf{D}$ can be formulated as

$$p(\mathbf{x}|\mathbf{D}) = \frac{p(\mathbf{D}|\mathbf{x})p(\mathbf{x})}{p(\mathbf{D})} \tag{1}$$