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The kern of non-homogeneous cross sections

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ABSTRACT

The state of stresses in non-homogeneous bars due to eccentric axial loads is examined. The governing formulae are written in coordinate free invariant forms by means of vector and tensor algebra. The concepts of centric and eccentric axial loads for non-homogeneous bars are introduced. A general solution is developed to obtain the kern of cross section of arbitrary shape. Examples show how the results derived can be applied to determine the kern of a given inhomogeneous cross section.

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1. Introduction

In dams, retaining walls or mansonry structures the designer is interested in determining the portion of cross section through which a compressive force can be applied without causing any tension at any point in the cross section. Such portion is called the kern (or core) of the cross section. The concept of kern has several applications in engineering practice [10,13–15,17]. The kern of a cross section is an old concept in mechanics of materials. This concept was first introduced by a French engineer Bresse in 1854 [8,11]. The concept of kernel of a cross section was widely used for pre-stressed concrete columns [9], beam supports [2], concrete dams [10] and a special case of S polygon [14,15,17]. Many textbooks and handbooks of mechanics of materials give a brief description on the concept of kern and they present the determination of the kern for very simple cross sections [7,12,18]. Wilson and Turcotte [16] gave a numerical method for determining the kern of general cross section. They approximate a general cross section with a polygon and then the kern of this polygon considered as an approximation of the kern of the original cross section. A paper by Mofid and Yavari [4] presents some theorems which describe the properties of the kern of a general cross section with arbitrary shape. Using these theorems the shape of the kern of a general homogeneous cross section can be obtained. Based on theorems of Mofid and Yavari [4] an algorithm is constructed in [5] which can be used to get the kern of any cross-section numerically. Paper [6] presents a very simple and practical analytical technique to determine the kern of an arbitrary cross section.

The present paper is concerned with non-homogeneous short prismatic bars loaded by axial load. By using a Bernoulli–Euler type beam theory a simple solution is obtained for combined extension (compression) and bending of bars with arbitrary cross section. The elastic modulus does not depend on the axial coordinate, i.e., the considered bar is axially homogeneous. This type of inhomogeneity is called cross sectional inhomogeneity. Vector-tensor formulation is used to analyse the problem of eccentrically loaded non-homogeneous bars. The formulation of the problem is given in coordinate free invariant form. A general solution is developed for arbitrary shape of the non-homogeneous cross section to obtain its kern. Papers [4–6,16] and books [2,7,8,12,18] did not use the vector-tensor formulations and they did not consider the non-homogeneous cross sections. The aim of this paper is to apply the formulation of paper [1] to short prism under the action of eccentric axial load. The concepts of centric and eccentric axial loads will be introduced and analysed for non-homogeneous bars. The equation of neutral axis (line of zero stress) and the kern of non-homogeneous cross section of arbitrary shape will be determined.

2. Formulation

Consider a straight bar of uniform cross section which has cross-sectional inhomogeneity. This means that the modulus of elasticity of the bar is the function of the cross-sectional coordinates x and y, so that $E = E(x, y)$. This function may be continuous or discontinuous. When the bar is made up of several laminates of

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different materials the modulus of elasticity for each laminate is constant but these constants may vary from laminate to laminate. In the case of functionally graded elastic materials, E is a smooth function of the cross-sectional coordinates x and y. Let the origin of the coordinate system $Oxyz$ be the E-weighted center of the left end cross section (Fig. 1) and let the planes of cross sections be parallel with the plane of axes x and y. In this case axis z connects the E-weighted centers of cross sections and we say that the axis (center-line) of the non-homogeneous bar is axis z and we have [1]

$$\int_A E(x,y)\boldsymbol{R}\,\mathrm{d}A = \boldsymbol{0}, \qquad \boldsymbol{R} = x\boldsymbol{e}_x + y\boldsymbol{e}_y. \tag{1}$$

In Eq. (1) A is the cross section of the non-homogeneous bar which is a bounded plane domain (Fig. 2), $E = E(x,y)$ is the Young modulus, and $\boldsymbol{e}_x, \boldsymbol{e}_y$ are unit vectors in x and y directions, respectively. The axial direction of the bar is denoted by the unit vector $\boldsymbol{e}_z$. The non-homogeneous bar is loaded by the axial loads $\boldsymbol{F}_1 = F\boldsymbol{e}_z$ and $\boldsymbol{F}_2 = -F\boldsymbol{e}_z$ whose points of applications L_1 and L_2 are in the end cross sections A_1 and A_2 (Fig. 1). According to Fig. 1 we can write

$$\overrightarrow{OL}_1 = \boldsymbol{\rho}_1 = a\boldsymbol{e}_x + b\boldsymbol{e}_y + l\boldsymbol{e}_z = \boldsymbol{\rho} + l\boldsymbol{e}_z, \qquad \overrightarrow{OL}_2 = \boldsymbol{\rho}_2 = \boldsymbol{\rho} = a\boldsymbol{e}_x + b\boldsymbol{e}_y. \tag{2}$$

The length of the bar is short, i.e., one so short that the effect of deflection is negligible. In the case of compressive axial force the stability problems are not considered. The stresses and strains are the same in every cross section. Therefore it is enough to consider only one cross section and to use only the cross-sectional coordinates x and y. The deformation of the bar – according to the beam theory of Bernoulli–Euler – can be described by the equation [1]

$$\varepsilon_z = \varepsilon_0 + \kappa\eta. \tag{3}$$

In Eq. (3) ε_z is the normal strain at point $P(x,y)$ and ε_0 is its value at the origin and κ is the curvature of the fiber whose points coincident with axis z before deformation. The normal vector of the planes of bent fibers is the unit vector $\boldsymbol{n}$ and $\boldsymbol{m} = \boldsymbol{e}_z \times \boldsymbol{n}$ (Fig. 2). In this case we have [1]

$$\eta = \boldsymbol{m}\cdot\overrightarrow{OP} = \boldsymbol{m}\cdot\boldsymbol{R}. \tag{4}$$

In the equations written above cross denotes the vectorial product of two vectors $\boldsymbol{e}_z$ and $\boldsymbol{n}$ while the scalar product of $\boldsymbol{m}$ and $\boldsymbol{R}$ is denoted by dot. The application of Hooke's law leads to the formula of normal stress σ_z

$$\sigma_z = E\varepsilon_z = E(\varepsilon_0 + \kappa\eta). \tag{5}$$

The resultant of normal stresses is as follows

$$F = \varepsilon_0\int_A E\,\mathrm{d}A + \kappa\int_A \eta E\,\mathrm{d}A. \tag{6}$$

Let us put the origin of the cross-sectional coordinate system Oxy to the E-weighted center of the cross section ($O = C_E$, Fig. 2). In this case we get [1]

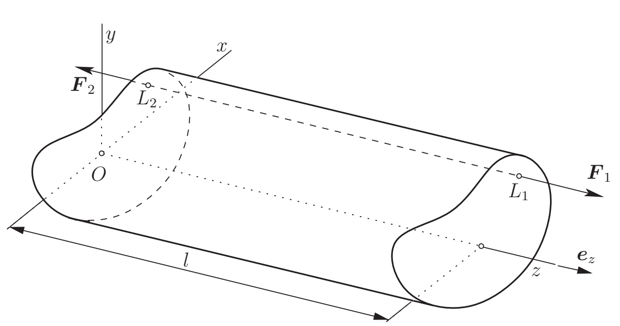

Fig. 1. Non-homogeneous bar with axial load.

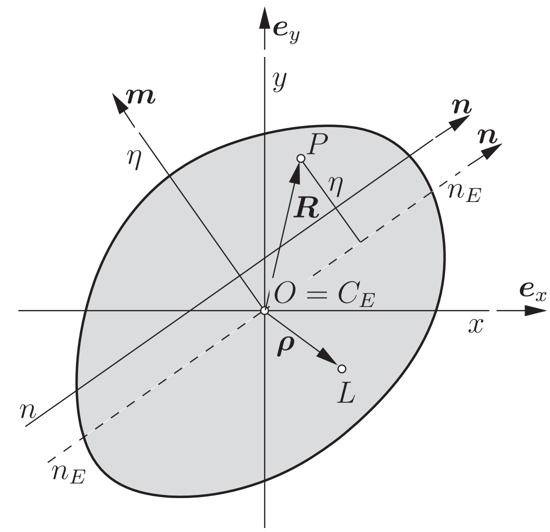

Fig. 2. The cross section of a non-homogeneous bar.

$$\int_A \eta E\,\mathrm{d}A = 0. \tag{7}$$

The combination of Eq. (6) with Eq. (7) gives

$$\varepsilon_0 = \frac{F}{S}, \quad S = \int_A E\,\mathrm{d}A. \tag{8}$$

The definition of the stress vector $\boldsymbol{t}_z$ yields the result

$$\boldsymbol{t}_z = \sigma_z\boldsymbol{e}_z = E\varepsilon_0\boldsymbol{e}_z + E\kappa(\boldsymbol{m}\cdot\boldsymbol{R})\boldsymbol{e}_z = E\varepsilon_0\boldsymbol{e}_z + E\kappa(\boldsymbol{e}_z\cdot(\boldsymbol{n}\times\boldsymbol{R}))\boldsymbol{e}_z, \tag{9}$$

i.e.,

$$\boldsymbol{t}_z = E\varepsilon_0\boldsymbol{e}_z + E\kappa\boldsymbol{n}\times\boldsymbol{R}. \tag{10}$$

The bending moment about point $O = C_E$ is caused by the axial load $\boldsymbol{F} = F\boldsymbol{e}_z$ whose point of application $L(a,b)$ is as follows (Fig. 2)

$$\boldsymbol{M} = \overrightarrow{OL}\times F\boldsymbol{e}_z = F\boldsymbol{\rho}\times\boldsymbol{e}_z. \tag{11}$$

The bending moment vector $\boldsymbol{M}$ can be computed in terms of stress vector $\boldsymbol{t}_z$ as

$$\boldsymbol{M} = \int_A \boldsymbol{R}\times\boldsymbol{t}_z\,\mathrm{d}A = \varepsilon_0\left(\int_A E\boldsymbol{R}\,\mathrm{d}A\right)\times\boldsymbol{e}_z + \kappa\int_A E\boldsymbol{R}\times(\boldsymbol{n}\times\boldsymbol{R})\,\mathrm{d}A. \tag{12}$$

Since $O = C_E$, we have

$$\boldsymbol{M} = \kappa\int_A E\boldsymbol{R}\times(\boldsymbol{n}\times\boldsymbol{R})\ \mathrm{d}A. \tag{13}$$

A simple calculation gives

$$\int_A E\boldsymbol{R}\times(\boldsymbol{n}\times\boldsymbol{R})\ \mathrm{d}A = \left(\int_A E(\mathbf{1}R^2 - \boldsymbol{R}\circ\boldsymbol{R})\ \mathrm{d}A\right)\cdot\boldsymbol{n}. \tag{14}$$

In Eq. (14) $\mathbf{1}$ is the second order two-dimensional unit tensor and the circle between two vectors denotes their tensorial (dyadic) product. We introduce the concept of bending rigidity tensor of the cross section with the following formula

$$\mathbf{J} = \int_A E(x,y)(\mathbf{1}R^2 - \boldsymbol{R}\circ\boldsymbol{R})\,\mathrm{d}A. \tag{15}$$

We note that, in paper [1] $\mathbf{J}$ is called the E-weighted inertia tensor of non-homogeneous cross section A about point C_E. Using Eq. (15) we can write

$$\boldsymbol{M} = \kappa\mathbf{J}\cdot\boldsymbol{n}. \tag{16}$$

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