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Relationships between internal forces, bracing patterns and lateral stiffnesses of a simple frame

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ABSTRACT

For a braced frame it is important to understand the relationships between the lateral stiffnesses, internal forces and bracing patterns in order to achieve an efficient design. This paper studies these relationships based on a four-bay and four-storey braced frame using both hand and computer analyses. Assuming that two bracing members can be placed arbitrarily in each of the four stories, there are 331,776 possible bracing arrangements. The best and the worst patterns of the total patterns are reviewed, with their characteristics summarized. All possible 256 symmetrically braced frames are further studied, allowing a more detailed examination of these relationships. Criteria for selecting bracing panels and bracing orientations are suggested based on the findings of this study and basic concepts, which may be applicable to the design of other braced frames. It is also found that the mega X brace or the double inversed V brace is the stiffest brace pattern, dependent on the aspect ratio of the braced panel and the size ratio of the columns to beams. This finding is confirmed using evolutionary structural optimization.

1. Introduction

Bracing systems provide the lateral stiffness for stabilising structures. They are widely used in multi-storey and high-rise buildings [1], temporary grandstands [2] and scaffolding [3]. In braced frames, appropriate bracing arrangements will increase lateral resistance and reduce internal forces, in particular bending moments in columns and girders. Such behavior also leads to a reduced consumption of materials. According to Gunel's summary [4], the maximum number of economic storeys in a tall building can be increased from 30 storeys in rigid frames to about 50 storeys in braced frames, and increased to over 100 by using braced tube forms. This leads to the question of what is the best bracing pattern for tall buildings in which the number of storeys is larger than the number of bays, and for temporary grandstands where the number of bays is larger than that of storeys.

The commonly used braced frames can be divided into three categories, concentrically-braced frames (CBFs), eccentrically-braced frames (EBFs) and knee-braced frames (KBFs) [5]. CBFs are most widely used due to their practical and economic advantages for conventional structures. There are also bracing variations used for earthquake resistant designs, such as buckling restrained braces (BRB) [6], shape memory alloy braced frame (SMABR) [7] and a BRB–SMA dual system [8]. This paper studies effective arrangement of concentric brace members for making a stiffer structure. The effective arrangement of bracing members in a frame structure is a topology problem which has been studied using two distinct approaches. One is computer based optimization and the other is based on structural concepts.

Optimization algorithms have been developed to find the optimum member sizes for braced frames [9] considering practical code and specification constraints [10]. However, the optimization of member sizes, in a defined bracing system, does not significantly improve structural efficiency. The more effective way is perhaps to adopt appropriate bracing patterns. Common brace configurations in practice include X, diagonal, V (or inverse V), K and Knee bracings. Comparison of structural behaviors of the well-known bracing configurations improved the understanding on the relationship between frame performance and brace layout. Maheri and Sahebi [11] experimentally examined the in-plane shear resistance of a bare RC frame panel, an X braced panel and a diagonally braced panel experimentally, and found that the resistance of the braced panels was 2.4 (diagonal) and 4.0 times (X brace) that of the bare frame panel. Sarno and Elnashai [5] compared the retrofitting effects of CBFs (X bracing) and mega-X-braced frames. The results showed that the later was more effective, with a 50% reduction in lateral drift and requiring about 20% less bracing steel. Using genetic algorithms, Kameshki and Saka [12] conducted a

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least weight optimization with code constraints and concluded that X, V and Z braces ranked the weight order from the least to the most. The finding was that the optimized V and Z braces were under stiffness control while the X brace was under strength control, providing further evidence of the higher efficiency of the X bracing. Turker and Bayrakstar [13] carried out both experimental and numerical investigations on the effects of brace configurations on the dynamic properties of frames. It was found that the tested cross braced frame (X) had the highest fundamental frequency, followed by the K braced frame. The frame with V, or inverse V, braces had the lowest natural frequency (about 75% of the X frame). Concentrating on the X bracing system, Moon [14] proposed a stiffness-based design method for preliminary estimation of the member sizes of steel braced tube structures. As part of the study, the empirical relationships between the number of duplicated modules, the tower aspect ratio and the optimum slope (between 40° to 50°) of the X brace were provided.

As a more systematic approach, topology optimization has been widely used to find the optimal bracing layout in frame structures. Investigations were conducted using discrete optimization methods on truss-like frame structures [15,16] but more attempts were made on continuum optimization as a more general approach [17–19]. With gradual removal of inefficiently used materials, Evolutionary Structural Optimization (ESO) was proposed by Xie and Steven [18] and soon became a simple and effective tool for optimizing bracing layouts. Liang et al. [19] revised the stiffness based ESO removal criteria from strain energy density to a performance index, considering both material efficiency and structural performance. They then applied this method to obtain optimal bracing topologies of two multi-storey steel frames. Huang and Wang [20] further extended the optimal study of bracing layout for seismic conditions. The concept of mean thickness was also proposed in order to solve the problem that the undersigned domain for bracings was not the real one. Using the Reuss and Voigt mixing rules, Mijar et al. [17] explored the optimum bracing layouts of a two-bay and six-storey frame to minimize structural compliance and maximize natural frequencies. In spite of the significant progress achieved, there are still limitations of the studies: some bracing layouts from optimization did not look intuitively good as there were no bracing members in the top storey [17,19]; the optimum bracing patterns were rather different from the normal configurations used in practice; the continuum topology optimization solutions are difficult to be converted into practical discrete brace structures; finally, limited by the optimization methods themselves, most of the obtained bracing layouts have not been proved to be the globally optimal results [21].

Still focusing on topology optimization, Stromberg et al. [22] analytically derived the optimum intersection point of a non-uniform X bracing pattern with the lower part taking three quarters of the full height. Symmetric conditions were used in the derivation, which was reasonable and illustrative for simplifying the problem. They also found that the simple optimized module could be extended to a frame with multiple modules. This promising finding showed that the study of basic unit could also be useful to more complicated frame cases. More interestingly, the derived bracing layout was practical and looked elegant. The authors further numerically studied the effect of column sizes on the optimal location of the intersection of X braces and provided useful information on the mechanism (controlled by shear action or overturning action). Zegard et al. [23] provided more detailed analytical derivation of the optimal crossing location of the non-uniform X brace. The optimal crossing height for 2D problems was 3/4 of the full frame height under constant stress level; while the optimum ratio for 3D problems was 5/8 for minimizing the structure weight and was 0.677 for maximizing the structural stiffness, because the fully stress condition could not be satisfied. These findings further improved the understanding of the optimal braced layout for possible variations of an X braced frame. The authors also more practically studied the effect of column size by considering vertical loads and summarized the optimal cross location in frames with multiple bay and storeys. However, the argument that the optimal ratio of 3/4 was still valid in a single bay frame with any magnitudes of the vertical loads may be questionable, because normally the crossing point would approach to 0.5H when the column size is big enough. Besides, if the optimum is evaluated via performance divided by material assumption, for example K(stiffness)/V(volume), or $1/[U$(strain energy)$V]$, the structural layout would be slightly different from that demonstrated by the authors. Finally, it should be noted that the studies by both Stromberg and Zegard were based on the constant stress assumption, which in many cases could be difficult to be satisfied in practice.

The other route for designing bracing systems is based on engineering concepts, which provide an intuitive understanding of bracing layout, not necessarily the best, but often efficiently meeting the engineering needs in practice. The dynamic tests of over 50 temporary grandstands, conducted by the Building Research Establishment of UK, showed that these grandstands had very low natural frequencies in both lateral and front-to-back directions. This triggered a study for improving the dynamic performance of these structures. Ji and Ellis [2] studied the effective bracing systems of temporary grandstands and concluded that bracing systems should be arranged following the concept of direct force paths. Based on this concept, five criteria were proposed for arranging bracing members for temporary grandstands. Ji [24] further studied concepts for designing stiffer structures. Based on a formula in textbooks, which had a history of over 150 years [25], three structural concepts were derived and presented in a memorable manner: (1) the more direct the internal force path, the stiffer the structure; (2) the more uniform the distribution of the internal forces, the stiffer the structure; and (3) the smaller the internal forces, the stiffer the structure. The concepts well reflected the pioneering works conducted by Maxwell and Michell, who proved that there was equivalence between the maximum structural stiffness (smallest compliance), the smallest deflection and the direct load paths under the condition of constant stress [22,23]. Numerical results such as those using ESO also agreed well with the concepts by showing that the stress based design (uniformly distributed stress) is equivalent to the stiffness based design (maximum stiffness) [26]. As a different point, it should be noted that the term "load path", following the definition of Maxwell and Michel, was represented by $\sum P_i l_i$ [27], reflecting the combined effect of the magnitude of internal forces in a member and the distance the forces travel, while in the current work the load path represents purely the internal force traveling distance. The separation of the two better reflect the mechanism governing the effective brace layout.

The three concepts provided a theoretical basis for designing the bracing layout of frame structures. Ji showed that the X bracing system based on these concepts led to a stiffer and more economical design than that obtained using a continuum optimization approach based on the same frame example [18,24]. A hand calculation showed that the lateral stiffness of a mega X braced four-bay four-storey frame is nearly four times that of the same frame braced with two groups of vertically arranged parallel diagonals (Fig. 1a). Physical models of the two calculated cases were constructed in order to demonstrate the effect of these bracing patterns on lateral stiffness, as shown in Fig. 1a [24,28]. An experimental study was conducted examining the lateral stiffness of three four-bay and four-storey frames with different bracing arrangements (Fig. 1b). The measurements of the maximum lateral displacements verified the theoretical predictions. Although the structures studied were simple, they provided an insight understanding and verified the