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Force-based higher-order beam element with flexural–shear–torsional interaction in 3D frames. Part I: Theory

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ABSTRACT

An innovative higher-order beam theory, capable of accurately taking into account flexural–shear–torsional interaction, is originally combined with a force-based formulation to derive the corresponding finite element. The selected set of higher-order deformation modes leads to an explicit and direct interaction between three-dimensional shear and normal stresses. Namely, cross-sectional displacement and strain fields are composed of independent and orthogonal modes, which results in unambiguously defined generalised cross-sectional stress-resultants and in a minimisation of the coupling of equilibrium equations. On the basis of work-equivalency to three-dimensional continuum theory, dual one-dimensional higher-order equilibrium and compatibility equations are derived. The former, which govern an advanced form of beam equilibrium, are strictly satisfied via stress fields arising from the solution of the corresponding systems of coupled differential equations. The formulation, which is numerically validated in a companion paper for both linear and nonlinear material response, inherently avoids shear-locking and accurately accounts for span loads. Finally, the superiority of force-based approaches over displacement-based ones, well established for inelastic behaviour, is also demonstrated for the linear elastic case.

1. Introduction

1.1. Review of higher-order beam theories

The geometrical features of many structural engineering elements make it possible to construct a set of governing differential equations which are considerably easier to solve than their complex three-dimensional continuum counterparts. In particular, beam theory is the simplest and simultaneously one of the most widely employed structural mechanics theories. Classical beam theory was initially based on the plane sections assumption, which was stated by Hooke in the XVIIth century. Further developed in the XVIIIth century by Bernoulli and Euler, such classical beam theory is only applicable to slender beams since it neglects the effect of shear deformation. It would not reach generalised engineering application until the end of the XIXth century. Meanwhile, with a view to the application to thick or deep beams, Rankine [1] and Bresse [2] included the relaxation of the restriction on the angle of shearing deformation, allowing the cross-section not to remain perpendicular to the beam's centroidal line, despite remaining a plane section and rigid in its own plane. Following the work by Timoshenko [3,4], this theory eventually was named after him.

It is well known that such classical beam theories, namely the Euler–Bernoulli and Timoshenko ones, are often not sufficiently accurate to predict the global member response and its internal stress–strain state. For instance, in the Timoshenko beam theory (TBT), the shear strain distribution is incorrectly assumed to be constant throughout the beam height; considering a simple rectangular cross-section, it does not respect the zero shear strain and stress boundary conditions at its top and bottom. Therefore, a shear correction factor is required to accurately determine the strain energy of deformation. Mindlin and Deresiewicz [5] computed such correction factor for a variety of beam cross-sections. Cowper [6], based on a pioneering integration of the three-dimensional equilibrium equations to form beam governing relations, obtained a new definition for the shear coefficient and derived expressions for homogeneous, isotropic symmetric cross-sections—see also Cowper [7]. An account of the early history of the shear correction factor can be found in Kaneko [8]. Research

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on this field has continued throughout the following decades (e.g., Hutchinson and Zillmer [9]; Renton [10]; Hutchinson [11]) and up to the present day (Dong et al. [12]). Within the framework of this paper, classical beam theories are considered to be of the first-order, i.e., the cross-sectional displacement fields are linear functions on each of the cross-sectional coordinates.

However, shear deformation effects are best considered through higher-order beam theories (HOBTs), wherein the axial displacement field is represented by a power series expansion in the cross-sectional coordinates, thus relaxing the constraint in the cross-sectional warping. Therefore, out-of-plane displacements of the cross-sectional points are allowed by using shape functions for the cross-sectional axial displacements which are at least quadratic in one coordinate or bilinear in both. Planar beam theories can be found in the literature (Stephen and Levinson [13]) which are similar in form to the TBT but account also for "shear curvature" and "transverse direct stresses", using those authors' nomenclature. The early work by Soler [14], wherein Legendre polynomials were used for thick rectangular elastic isotropic beams, as well as orthotropic beams (Tsai and Soler [15]), should be mentioned. This family of polynomials was employed because their completeness, convergence and orthogonality properties are well formulated. Furthermore, the usual stress-resultants of classical beam theory appear naturally. Even without previous knowledge of such approach, similar reasons also guided the use of Legendre polynomials in the theoretical developments of the present study.

Levinson [16] used a third-order beam theory satisfying zero shear strain conditions at both the upper and lower edges of the beam, obviating the need for the shear coefficient. The equations of motion therein derived are not variationally consistent, which was later corrected by other authors, either making use of Hamilton's principle (Bickford [17]) or the principle of virtual displacements (Reddy [18]). The variational consistency of Bickford's theory does not necessarily seem to imply, however, superior accuracy (Rychter [19,20]). The Euler–Lagrange equations of motion in Bickford's theory are typically displayed in terms of displacements, mechanical parameters (describing a linear elastic constitutive relation), and cross-sectional geometric properties (Petrolito [21]). Nevertheless, it is naturally possible to express them in a format wherein specific constitutive relations are not yet assumed (Reddy [22]). Such arrangement has the advantage of showing immediately the generalisation of stress-resultants that is required in higher-order theories. For example, in Bickford's theory the common definition of shear force gives place to a new definition of a higher-order shear force, involving the cross-sectional integral of the shear stresses; additionally, a higher-order moment of the normal stresses also shows up. It should be pointed out, however, that it is possible to construct HOBTs—such as the one herein proposed—wherein the classical definitions of the stress-resultants are preserved.

Based on Bickford's theory, a two-node beam finite element with three degrees of freedom per node was later developed and tested (Heyliger and Reddy [23]). Approximately at the same time, Kant and Manjunatha [24]—and later Manjunatha and Kant [25]—proposed beam theories with kinematic fields having different orders of variation for both the longitudinal and transverse displacements; the authors used Lagrangian four-noded cubic elements with different number of degrees of freedom per node (ranging from three to seven, according to the complexity of the underlying theory).

The Lo–Christensen–Wu theory (Lo et al. [26,27]) is an elegant theory that is widely used by researchers for the analyses of shear deformable beams and plates; it expands the axial displacement field as a cubic function in the thickness coordinate, while the polynomial expansion for the transverse displacement is truncated at one order lower. Vinayak et al. [28] and Prathap et al. [29] carry out a systematic evaluation of the Lo–Christensen–Wu theory, comparing the results of finite element analyses with available closed-form classical and elasticity solutions.

The refined model by Kim and White [30], developed for both thin- and thick-walled composite beams, is of interest since it accounts for transverse shear effects of the cross-section and of the beam walls, as well as primary and secondary warping. Rand [31] devised a model to handle arbitrary solid cross-sections or general thin-walled geometries; it considers five degrees of freedom, namely three cross-sectional displacements, a twist angle and a 3D warping function. The importance of the latter, which is made dependent on the boundary conditions (unlike traditional beam theories), is demonstrated in a subsequent study by the same author [32].

The number of proposals associated with composite beam modelling is countless and can be found in the literature reviews of Ghugal and Shimpi [33] and Volovoi et al. [34]. Nevertheless, more recent works deserve to be mentioned. In particular, the variational asymptotic beam sectional analysis (VABS) suggested by Yu et al. [35] is of relevance; therein, instead of assuming a 3D warping displacement, they compute it in terms of the 1D generalised strains. Also, in the context of the use of trigonometric functions, a new three-noded beam finite element was recently conceived by Vidal and Polit [36] for the analysis of laminated beams.

A significant improvement over previous HOBTs was accomplished on the so-called 'Carrera's Unified Formulation' (Carrera and Giunta [37]), also known as CUF, by allowing the order of the theory and, consequently, the number of cross-sectional displacement modes it takes into account, to be a free parameter. In view of the similarities to the current work and the additional fact that it will be considered for comparison purposes, a short introductory note on such formulation should be made. Originally applied to the modelling of anisotropic plate and shell structures (Carrera [38]), the method proposes a systematic manner of formulating axiomatically refined beam models by choosing the desired order of the theory. Using a concise notation for the kinematic field, the governing differential equations and the corresponding boundary conditions (BCs) are reduced to a 'fundamental nucleo' in terms of the displacement components, which does not depend upon the approximation order. The finite element formulation of the CUF for beam structures (Carrera et al. [39]) includes two, three and four-noded elements—using respectively linear, quadratic and cubic approximations along the beam axis—with different higher-order models for the cross-section displacement field. The displacement components are expanded in terms of the cross-section coordinates using Taylor-type expansions. The effectiveness of higher-order terms in the context of the CUF is analysed in a subsequent work (Carrera and Petrolo [40]), while its applicability to the free vibration of rotating beams is carried out in a new study (Carrera et al. [41]). A compilation of the beam formulations and results obtained with the CUF was recently published (Carrera et al. [42]).

1.2. Finite element formulations and objective of the study

In the context of finite element formulations for solid mechanics, it is well-known that low-order elements in classical displacement approximations lead to unsatisfactory performance, which can be due to either locking in the incompressible limit or to poor accuracy (namely in bending-dominated behaviour). The use of energy functionals representing multi-field variational principles provide a natural setting for the formulation of mixed finite element methods, and an approach to by-pass the aforementioned problems. In a mixed method it is possible to independently approximate all fields that exist in the functional, which opens the door to interesting methods of analysis. In particular, the three-field formulation proposed by de Veubeke [43,44] and commonly known as Hu–Washizu [45,46] allows to approximate the displacement, stress and strain as independent variables (see also