Engineering Structures 88 (2015) 111–124

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Multiobjective robust optimization for crashworthiness design of foam filled thin-walled structures with random and interval uncertainties

Fangyi Li [a], Guangyong Sun [b,*], Xiaodong Huang [b], Jianhua Rong [a], Qing Li [c]

[a] School of Automotive and Mechanical Engineering, Changsha University of Science and Technology, Changsha 410114, China
[b] State Key Laboratory of Advanced Design and Manufacture for Vehicle Body, Hunan University, Changsha 410082, China
[c] School of Aerospace, Mechanical and Mechatronic Engineering, The University of Sydney, Sydney, NSW 2006, Australia

ARTICLE INFO

Article history:
Received 8 February 2014
Revised 15 January 2015
Accepted 18 January 2015
Available online 11 February 2015

Keywords:
Aluminum foam
Crashworthiness
Multiobjective optimization
Robustness design
Hybrid uncertainties

ABSTRACT

To improve crashing behavior of aluminum foam-filler columns design optimization has proven rather effective and been extensively used. Nevertheless, an optimal design could become less meaningful or even unacceptable when some uncertainties present. Parametric uncertainties are often treated as random variables in conventional robust optimization. Taking foam filled thin-walled structure as an example, which could also exhibit probabilistic and/or bounded nature of uncertainties, it may be more appropriate to describe them with hybrid uncertainties by using random variables and interval variables. Furthermore, evaluation of product quality often involves a number of criteria which may conflict with each other. To address the issue, this paper presents a multiobjective robust optimization to explore the design problems of parametric uncertainties involving both random and interval variables in foam filled thin-walled tube, in which specific energy absorption (SEA) and peak crushing force are considered as the design objectives and the average crash force is considered as the design constraint. A nesting optimization procedure is proposed here to solve the multiobjective robust optimization problem. In the outer loop, the Non-dominated Sorting Genetic Algorithm II (NSGA-II), is implemented to generate robust Pareto solution. In the inner loop the Monte Carlo simulation is performed to evaluate the impact responses of the mixed uncertainties to the robustness of optimized design. The example demonstrates the effectiveness of the proposed robust crashworthiness optimization involving both random and interval variables.

1. Introduction

Nowadays various metallic foams have been widely used in energy absorber attributable to their excellent capacity of energy absorption, extraordinary light weight and relatively low cost in automotive, aerospace, defense and other industries [1,2]. To understand the characteristics of energy absorption, Hanssen et al. [3] carried out comprehensive experimental and numerical studies on the foam filled thin-walled aluminum columns under axial crushing. Through a comparative study, it is suggested that foam-fillers could be used to increase the effectiveness of energy-absorption considerably [4].

Design optimization, aiming to maximize the performance of thin-walled structures filled with foam or cellular materials, has drawn increasing attention recently. For example, Nariman-Zadeh et al. [5] used multiobjective genetic algorithm to minimize the weight and maximize the energy absorption of square aluminum column with aluminum foam-filler. Zarei and Kröger [6] used the multicriteria design optimization (MDO) to optimize the crashworthiness of foam-filled tube. Hou et al. [7] explored single and multiple crashworthiness criteria for optimizing foam filled structures. Shariati et al. [8] performed the design optimization of spot-welded column with foam filler. Bi et al. [9] investigated single-cell and triple-cell hexagonal columns filled with aluminum foams for maximizing specific energy absorption. Sun et al. [10] proposed to seek an optimal density variation for functionally graded foam configuration to improve crashworthiness by using single and multiobjective optimization approaches. More recently, Zhang et al. [11] explored the design issues of thin-walled bitubal structures filled with aluminum foam and sought the optimal solutions for improving the crashworthiness by using the genetic algorithm (GA) and Non-dominated Sorting Genetic Algorithm II (NSGA II), respectively.

These abovementioned studies on thin-wall structures filled with foam materials are restricted on deterministic optimization, where it is assumed that all the design variables and parameters involved are certain [12]. It remains unclear, however, how the

* Corresponding author. Tel.: +86 731 8882 1717; fax: +86 731 8882 2051.
E-mail address: sgy800@126.com (G. Sun).

http://dx.doi.org/10.1016/j.engstruct.2015.01.023
0141-0296/

uncertainty could affect the optimization process and design outcomes. Ideally, minimum influences would be expected from a perspective of design robustness. For this purpose, robust optimization is of more practical implication which could yield the least sensitivity to presence of uncertainties [13–15]. While most of existing robust design problems have concerned with single objective optimization, real-life problems often involve a number of quality and/or performance indices and some of them could be conflicting with each other. Recently, some researchers have conducted multiobjective robust optimization for crashworthiness problems. For example, Sinha et al. [16] presented a response surface based multiobjective crashworthiness optimization for side impact and followed up with multiobjective robust optimization subject to mean-effective value and additional robustness constraints. Zhu et al. [17] systematically studied design of automotive front-body structure based on robust optimization involving the uncertainties of design variables with sheet gauge and yield limit of materials. Sun et al. [18] explored multiobjective robust optimization to address the effects of parametric uncertainties on multiple crashworthiness criteria, where several different levels of sigma (standard deviation) criteria were considered to explore the effects of the variations. Lönn et al. [19] performed the robust crashworthiness design for a vehicle bumper using the meta model based Monte Carlo simulations. Khakhali et al. [20] carried out the robust multiobjective optimization for energy absorption and peak force criteria in the S-shaped box beam. More recently, Gu et al. [21] conducted a comparative study on multiobjective reliable and robust optimizations for crashworthiness design of vehicle structures.

In the abovementioned robust design cases, the probability distributions for inputs are assumed to be known. It is, however, often difficult (if not impossible) to obtain sufficient data for determining probability density functions in most of real-life problems, making the use of interval model attractive. In the interval method [22–24], only the lower and upper bounds of the uncertain parameters are required, unnecessarily knowing their precise probability distributions. Nevertheless, limited studies on interval variables based multiobjective robust optimization have been conducted to date. For example, Soares et al. [25] introduced the interval robust multiobjective optimization that allowed to find an enclosure of the robust Pareto frontier through the bounded intervals, in which the best performance can be achieved for the worst case scenario. Chen and Wu [23], Gunawan and Azarm [26] and Hu et al. [27] adopted the interval uncertainty to account for robustness for both objective and constraint functions through a two-level approach. Li and Azarm [28] proposed a multiobjective collaborative robust optimization (MCRO) approach by employing multiobjective genetic algorithm, in which uncertain parameters are constructed using interval and MCRO models. Li and Li [29] also presented crashworthiness design of vehicle by optimizing the objective robustness via an interval approach.

Note that in many real-life applications, some uncertainties are random and allowed to realistically estimate probability distribution, whilst others are only uncertain-but-bounded due to limited information or lack of knowledge. As a result, uncertain variables are typically classified as random variables and interval variables based upon the availability of probability distribution [30–32]. Du et al. [33] performed the assessment of robustness and proposed a method to synthesize random and interval variables, and late they [34] extended robustness assessment to multidisciplinary systems involving both random and interval variables. Gao et al. [35] investigated the mean and standard deviation of random interval structural responses by combining the Taylor expansion, matrix perturbation and random interval moment techniques.

It is noted that the existing studies available in literature have focused on solving the single objective optimization with the combination of random and interval variables. To the authors' best knowledge, there has been no report available yet concerning multiobjective robust crashworthiness involving random and interval variables for foam filled thin-walled structure despite its strong practical implication. This paper aimed to tackle this problem by exploring the robustness involving mixed uncertain variables, potentially providing new insights into the robust multiobjective optimization of foam-filled crushing structures.

2. Multiobjective robust optimization methods

2.1. Formulations of multiobjective optimization problem

Without considering randomness of design variables and system parameters, a deterministic multiobjective optimization problem can be formulated mathematically as,

$$\begin{cases} \min_{\mathbf{d}} f_m(\mathbf{d}), m = 1, 2, \ldots, M \\ \text{s.t.} \begin{cases} g_l(\mathbf{d}) \leqslant 0, & l = 1, 2, \ldots, L \\ h_s(\mathbf{d}) = 0, & s = 1, 2, \ldots, S \\ \mathbf{d}^L \leqslant \mathbf{d} \leqslant \mathbf{d}^U, \\ \mathbf{d} = [d_1, \ldots, d_H]^T \end{cases} \end{cases} \tag{1}$$

where $f_m(m = 1, 2, \ldots, M), g_l(l = 1, 2, \ldots, L)$ and h_s $(s = 1, 2, \ldots, S)$ are the objective, inequality and equality constraint functions, respectively. $\mathbf{d}^L$ and $\mathbf{d}^U$ denote the lower and upper bounds of design variable $\mathbf{d}$, respectively.

With both random and interval variables involved in the design variable $\mathbf{d}$, the corresponding multiobjective optimization problem can be revised as:

$$\begin{cases} \min_{\mathbf{d}} f_m(\mathbf{x}, \mathbf{a}), m = 1, 2, \ldots, M \\ \text{s.t.} \begin{cases} g_l(\mathbf{x}, \mathbf{a}) \leqslant 0, & l = 1, 2, \ldots, L \\ h_s(\mathbf{x}, \mathbf{a}) = 0, & s = 1, 2, \ldots, S \\ \mathbf{d}^L \leqslant \mathbf{d} \leqslant \mathbf{d}^U, \\ \mathbf{d} = \{\mathbf{x}, \mathbf{a}\}^T \end{cases} \end{cases} \tag{2}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_I)^T$ is the vector consisting of I random variables and $\mathbf{a} = (a_1, a_2, \ldots, a_P)^T$ is the vector consisting of P interval variables. Let $\mathbf{a}_l$ and $\mathbf{a}_u$ be the lower and upper bounds of $\mathbf{a}$, $\mathbf{a}$ resides over its interval $[\mathbf{a}_l, \mathbf{a}_u]$ $(\mathbf{a} \in [\mathbf{a}_l, \mathbf{a}_u])$. Then the midpoint $\bar{\mathbf{a}}$ of interval vector $\mathbf{a}$ is given by

$$\bar{\mathbf{a}} = \frac{1}{2}(\mathbf{a}_l + \mathbf{a}_u) \tag{3}$$

Due to the effect of interval vector $\mathbf{a}$, the mean values and standard deviations of objectives can be thus represented in terms of the intervals [33].

2.2. Assessment of robustness

As a result of the co-existence of both random and interval variables, the mean values can be characterized in terms of intervals for each objective function. The average mean value of objective $f(\mathbf{x}, \mathbf{a})$ can be calculated as [33,34],

$$\bar{\mu}_f = \frac{1}{2}(\mu_f^{\max} + \mu_f^{\min}) \tag{4}$$

where $\mu_f^{\max}$ and $\mu_f^{\min}$ are the means of maximum and minimum values corresponding to the interval, respectively.

Due to the existence of intervals, the standard deviation σ_f of $f(\mathbf{x}, \mathbf{a})$ is also an interval for each objective function. In other words, σ_f is bounded by the values of maximum and minimum due to the interval $\sigma_f^{\max}$ and $\sigma_f^{\min}$. The robustness can thus be