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An improved four-parameter model with consideration of Poisson's effect on stress analysis of adhesive joints

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ABSTRACT

An improved four-parameter model considering Poisson's effect is presented to analyze interface stresses of adhesively bonded joints. The existing theoretical models of adhesive joints analyze the adhesive layer as a beam without considering Poisson's effect, which violates the Hooke's law and cannot satisfy the compatibility condition of the adhesive layer; furthermore, the bending moment of the adhesive layer is neglected by assuming the thin thickness of adhesive layer. To eliminate these flaws, the present study models the adhesive layer as a 2-D elastic continuum in which both the Hooke's law and equilibrium equations are fully satisfied. The longitudinal strain caused by the transverse stress is also considered, and it is proved to have a significant effect on the interface stress distributions. The present model regains the missing bending moment of the adhesive layer which is absent in the existing models, and it satisfies all the boundary conditions. The validity of the new model is demonstrated by its excellent agreements with the results from the numerical finite element analysis when predicting the interface stress distributions for the single-lap joints and CFRP-strengthened reinforced concrete beams, and the broader applicability of the present model can be expected. The new model developed presents explicit closed-form solutions for the interface stresses and beam forces, and it provides an accurate tool for design analysis of adhesively bonded joints or interfaces.

1. Introduction

Adhesively bonded joints are increasingly used in composite structures to connect components because of their many advantages over other joining methods (e.g., the mechanical bolted joints). However, a high stress concentration exists near the end of the adherend–adhesive interface because of either the material or geometrical mismatch or both, which accounts for the premature failure of designed structures due to debonding and peeling of the joint. Therefore, accurate prediction of the interface stresses is significant to avoid such premature failure. To this aim, numerous theoretical and experimental studies have been conducted. Goland and Reissner [11] modeled the adhesive layer as the continuously distributed shear and vertical springs (the so-called G–R model). In this model, both the shear and normal stresses were assumed to be invariant along the thickness direction, and the adhesive layer was modeled as a two-parameter elastic foundation. On this basis, some simple explicit closed-form expressions of interface stresses and beam forces were obtained by many researchers [7,21,24,23,28,8,1,15,5,6,9,27,12]. The interface stresses predicted by the two-parameter elastic foundation model reached good agreements with the finite element analysis (FEA) as a whole [25], and the maximum interface stresses were also captured. However, obvious discrepancy was observed near the edge of the adhesive layer.

To overcome this drawback, many refined models have been developed. The finite element analysis [25] revealed that the shear and peel stresses along the top interface (TA) are different from those of the bottom interface (BA), and the discrepancy grows with the increased thickness of the adhesive layer. Meanwhile, the stress distribution becomes non-uniform when the adhesive layer is not thin enough. To capture these non-uniform stress distributions accurately, a full elasticity analysis should be applied to the adhesive layer. Using this as a start point, a complete elastic model was proposed by Radice and Vinson [20]. In this model, the Airy stress potential was expanded into a polynomial series and the coefficients were obtained using the Rayleigh–Ritz theory. However, this elastic model is too complex and does not result in the

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explicit expressions for the interface stresses. Thus, it can be seen that it is difficult to find a complete elastic solution, and there is a need to develop simple yet accurate explicit solution of adhesively bonded interface models based on some reasonable assumptions.

To obtain the different peel stresses along different adherend–adhesive interfaces, a three-parameter elastic foundation model [2,30,31,32] was proposed by introducing the deflection of the adhesive layer as an additional parameter. By assuming the linear transverse normal stress variation and constant shear stress through the thickness of the adhesive layer, a closed-form high-order model for RC beams strengthened with FRP strips was also proposed [3,10,17,18,35]. Although the peel stresses along different adherend–adhesive interfaces were assumed to be different in these models to satisfy the force equilibrium conditions which are not met in the two-parameter model, the shear stress was still assumed to be invariant through the thickness direction, which contradicts with reality and makes the model incapable of capturing the shear stress concentration in a small region near the edge of the adhesive layer. In order to capture the functions of the shear and peel stresses along the adherend–adhesive interfaces, the analytical models [14,13] were developed by assuming the linear longitudinal and transverse displacement along the thickness direction. Even though different shear and peel stresses at the two adherend–adhesive interfaces were obtained, the equilibrium equations in the adhesive layer could not be satisfied. To better predict the interface stress distributions along the adhesive joints, the extensive effort has been devoted to the continuum analysis based on the assumption of the linear variation of longitudinal stress and the quadratic variation of the shear stress across the adhesive thickness. Shen et al. [22] characterized the interface stress as the trigonometric series, and the semi-analytical solution was obtained using the principle of the complementary energy. Yang et al. [33] extended this method to the adhesive joints subjected to arbitrary loadings. However, in order to make the solutions converged, thousands of trigonometric terms were required and a large amount of computational effort was made, which is thus inconvenient to be applied for design analysis of adhesive joints. In most recent studies, two analytical solutions [34,4] were developed by assuming the linear variation of longitudinal normal stress and the quadratic variation of shear stress in the adhesive layer. Based on the assumption of the linear variation of longitudinal stress which is a logic error in fact, Zhao and Lu [34] obtained the variation of longitudinal stress by using the equilibrium equations and Hooke's law. Although Chen and Qiao [4] recently obtained the shear and transverse normal stress distributions through the thickness of the adhesive layer, the relationship between the stress and deformation was not consistent and the adhesive layer was modeled as a beam without considering the Poisson's effect. In addition, the bending moment of adhesive layer was neglected by assuming the thickness of adhesive layer is thin enough and its elastic modulus is relatively small.

In this study, an improved four-parameter model of adhesive joints is proposed to study the stress distributions along the adherend–adhesive interfaces. The adhesive layer is modeled as a 2-D elastic continuum in which both the Hooke's law and equilibrium equations are satisfied. The deformation of the adhesive layer is obtained by using the constitutive equations instead of estimating the deformation with the interface compliance or flexibility coefficient. The missing bending moment of the adhesive layer is also retrieved, and all the force and moment boundary conditions are satisfied, which mean that this model can be applied to the case that the thickness of adhesive layer is in the same order or even a few more times thicker than that of the strengthening materials (e.g., the thin CFRP strengthening layer). To validate the present model, the comparisons among the present model with the numerical finite element analysis (FEA) are conducted. A parametric study is finally conducted to reveal the influence of the Poisson's ratio which is commonly neglected in most of existing studies.

2. Four-parameter, elastic continuum model of an adhesive joint

2.1. Adhesively-bonded layer-wise bi-layer beam system

Consider a beam system in which two adherends are bonded through a thin or moderately-thick layer of adhesive. These two adherends are modeled as the Timoshenko beams [26] with the thickness of h_1 and h_2 and the width of b_1 and b_2, to account for the shear deformation of the adherends, and they are bonded by the adhesive layer of the width and thickness of b and h_a, respectively. A simple and generic adhesively-bonded beam system configuration with the uniformly distributed load (UDL) and/or axial force at the end is considered, though the model can be applied to more complicated loading and boundary configurations.

The present interface stress analysis is based on the following five assumptions: (1) Each individual layer is elastic, homogeneous, and orthotropic; (2) There is a deformation compatibility condition between the layers, i.e., no slip or opening-up occurs at the interfaces; (3) The adhesive is considered to be in a plane stress state; (4) The longitudinal stresses are assumed to vary linearly across the adhesive layer thickness [22]; and (5) The vertical deformation of the middle plane of the adhesive layer (MA) is equal to the average vertical deformation of the whole adhesive layer.

In this study, the analysis is focused on the bonded interface area of the adherends. Consider a typical infinitesimal isolated body of the adhesively bonded interface which is illustrated in Fig. 1. For the shown free body diagram, the following equilibrium equations are established:

$$\frac{dN_1(x)}{dx} = b\tau_1(x), \quad \frac{dN_2(x)}{dx} = -b\tau_2(x) \tag{1}$$

$$\frac{dQ_1(x)}{dx} = b\sigma_1(x) + q, \quad \frac{dQ_2(x)}{dx} = -b\sigma_2(x) \tag{2}$$

$$\frac{dM_1(x)}{dx} = Q_1(x) - \frac{h_1}{2}\tau_1(x)b, \quad \frac{dM_2(x)}{dx} = Q_2(x) - \frac{h_2}{2}\tau_2(x)b \tag{3}$$

$$N_a = \int_{-\frac{h_a}{2}}^{\frac{h_a}{2}} b\sigma_x dz, \quad M_a = \int_{-\frac{h_a}{2}}^{\frac{h_a}{2}} b\sigma_x \cdot z dz, \quad Q_a = \int_{-\frac{h_a}{2}}^{\frac{h_a}{2}} b\tau_{xz} dz \tag{4}$$

where $N_i(x)$, $Q_i(x)$ and $M_i(x)$ ($i = 1, 2, a$) are the internal axial forces, transverse shear forces, and bending moments in the adherends and adhesive layers, respectively; $\sigma_1(x)$ and $\sigma_2(x)$ are the peel stresses along the top adherend–adhesive interface (commonly abbreviated as TA) and the bottom adherend–adhesive interface (commonly abbreviated as BA), respectively; $\tau_1(x)$ and $\tau_2(x)$ are the shear stresses along the TA and BA interfaces, respectively. Note that the overall equilibrium conditions in the bi-layer beam region of the structure under the uniformly distributed load (UDL) q shown in Fig. 1 require that:

$$N_1 + N_2 + N_a = N_{10} + N_{20} + N_{a0} = N_T \tag{5}$$

$$Q_1 + Q_2 + Q_a = Q_{10} + Q_{20} + Q_{a0} + qx = Q_T \tag{6}$$

$$\begin{aligned} M_1 + M_2 + M_a + N_1\frac{h_1 + h_2 + 2h_a}{2} + N_a\frac{h_2 + h_a}{2} \\ = M_{10} + M_{20} + M_{a0} + N_{10}\frac{h_1 + h_2 + 2h_a}{2} + N_{a0}\frac{h_2 + h_a}{2} \\ + Q_T x - \frac{1}{2}qx^2 = M_T \end{aligned} \tag{7}$$