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A higher order beam model for thin-walled structures with in-plane rigid cross-sections

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ABSTRACT

A higher order thin-walled beam model for the analysis of thin-walled structures considering the cross-section warping and shear deformation is presented in this paper. The formulation is derived from the corresponding elasticity governing equations by adopting an approximation of the beam displacement field over the cross-section by a set of linearly independent basis functions, which are refined according to the accuracy required for the representation of the three-dimensional behaviour of the structure. A set of beam-like equations is obtained through the integration over the cross-section of the corresponding elasticity equations, properly weighted by the cross-section approximation functions. A set of uncoupled beam deformation modes is obtained from a non-linear eigenvalue problem that stems directly from the homogeneous solution of the beam differential equilibrium equations, being reduced to a generalized eigenvalue problem for a transversely rigid cross-section. The criteria put forward for uncoupling the beam deformation modes is mathematically consistent and allows the interpretation of the involved structural phenomena. Two kinds of deformation modes are obtained: (i) classic deformation modes, being associated with a null eigenvalue, requiring an adequate computation of a Jordan chain and (ii) higher order modes corresponding to the non-null eigenvalues, allowing to measure the mode decay along the beam axis. Some examples are presented in order to verify the capability of the model to simulate the non classic effects associated with the higher order deformation modes of thin-walled structures.

1. Introduction

The analysis of thin-walled structures through a one-dimensional model requires an enrichment of the displacement field approximation on the beam cross-section in order to adequately capture the three-dimensional behaviour of the structure. Hence, and towards the effective use of a beam model in the analysis of thin-walled structures, a set of deformation modes, defining non-classic effects associated with the corresponding 3D structural behaviour, has to be defined for the cross-section. The successful development of a beam theory is therefore dependent on the definition of such higher order modes. Several semi-analytical beam formulations derived from the elasticity theory have successfully dealt with the problem through different approaches:

(i) The quantification of the Saint-Venant principle, as defined by Toupin [49] and Goetschel [20], provides a framework for developing a beam theory since it allows to represent the three-dimensional continuum mechanics in terms of the cross-section higher order modes (i.e., effects with a decaying behaviour). The higher order effects of a thin-walled beam model were obtained in Giavotto et al. [19], Bauchau [2], Morandinia [32], Genoese [12], Ferradi [17,18] and cast within the framework of a geometrically exact beam theory in Genoese [14,15]. Higher modes of an anisotropic beam were considered in Genoese [13].

(ii) The variational asymptotic method (VAM) developed by Berdichevsky [4] considers the 3D elasticity problem decomposed into a one-dimensional formulation and a 2D problem over the cross-sectional plane, allowing to retrieve classic solutions and derive beam-like equations for higher order effects, Cesnik [9], Hodges [23], Yu [52].

(iii) A beam formulation obtained by considering the definition of the displacement field through Taylor expansions is derived within the framework of a so-called unified formulation (CUF), Carrera and Giunta [7], Carrera et al. [5], allowing to improve the corresponding solutions towards a more accurate representation of higher order effects due to its hierarchical nature, Carrera and Petrolo [6], being applied to beam models having a cross-section with an arbitrary geometry, Carrera et al. [8].

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An efficient procedure adopted to capture the 3D structural behaviour through a beam model corresponds to consider the approximation of the displacement field on the cross-section by a set of linearly independent basis functions. In general, the cross-section is divided into elements over which the displacement field is approximated, being a global approximation function (at a cross-section level) obtained by ensuring compatibility between elements. The three-dimensional elasticity problem is thus considered through a transverse analysis at the cross section level, being the corresponding accuracy dependent on the approximation of the displacement field over the cross-section.

For thin-walled structures, and since the cross-section behaviour is often represented with reference to the corresponding middle surface, 1D elements have been usually adopted for the discretization of the cross-section, being the displacement field approximated along the cross-section midline. Beam models based on a cross-section discretization that considers warping functions of thin-walled cross-sections were presented in Laudiero [29], Razaqpur and Li [41], Prokić [37], Razaqpur and Li [42], Prokić [38,39], Kim and Kim [25], Prokić [40], Kim and Kim [26], Saadé et al. [44]. A new finite beam model for the analysis of non-uniform torsion of beams that considers the cross-section warping through an additional degree of freedom and includes secondary torsional phenomena and shear effects has been proposed in Murin [34]. An alternative procedure, cast within the boundary element method, for the analysis of non-uniform torsion of bars considering secondary warping was put forward in Tsipiras [50], Sapountzakis et al. [45], Dikaros et al. [11].

A successful theory for the analysis of thin-walled structures capable of considering the cross-section warping is the so-called generalised beam theory (GBT). This theory has been developed from the seminal work of Schardt [46] towards its applicability to more generic cross-section midline geometries Möller [33], Simão [48], Dinis et al. [10], Hanf [22], Ranzi [43], Piccardo [36] and to variable cross-sections, Nedelcu [35]. However, in GBT formulations the approximation of the displacement field is bound to the cross section mechanical behaviour either by imposing unit displacements at specific nodes whilst enforcing others displacements to be null, and in compliance with deformation assumptions, or by imposing unit deformations at each wall while keeping the deformations of other walls null; e.g., to consider the out-of-plane displacements in closed cross-sections, the displacement field approximations of Gonçalves [16] considers the imposition of a unit displacement at independent "natural nodes" that have to be properly selected so as to comply simultaneously with (a) the null membrane shear hypothesis and (b) the compatibility condition at a node where more than two walls converge. To include shear deformation, an additional set of modes, "membrane shear deformation modes", is defined by imposing a unit shear distortion for each wall while enforcing (a) the axial displacements to be null at independent "natural nodes" and (b) the Vlasov's assumption in all the remaining walls.

Beam models that consider the discretisation of the thin-walled cross-section into two dimensional elements have also been successfully considered in Yoon [54], Høgsberg [24], Yoon [55].

The enrichment through an approximation scheme for the displacement field on the cross-section allows a beam model to reproduce the structural three-dimensional behaviour. However, for the beam model to be efficiently adopted, an uncoupling of the corresponding governing equations, which allows to identify uncoupled structural phenomena through the corresponding deformation mode, is a key feature. The uncoupling of the cross-section warping derived by Vlasov [51] relatively to the classic modes is performed by considering a change of the referential's origin so as to cancel coupled terms, corresponding to the definition of the cross-section shear centre. However, for beam formulations that consider sets of additional deformation modes, the criterion for uncoupling such modes is not conveniently addressed. In fact, several innovative and ingenious forms of obtaining uncoupled models by considering some "*ab initio*" conditions together with orthogonalizations procedures that stem from eigenvalue statements. However, these eigenvalues problems are not directly connected with the solution of the beam governing equations, being instead aimed for the simultaneous diagonalization between pairs of the coefficient matrices of the corresponding governing equations, Sedlacek [47], Maisel [30], Schardt [46] and Razaqpur and Li [41]. This process has however the disadvantage of not being physically consistent, failing to obtain a set of a linear combination of deformation modes capable of reproducing the beam governing solutions; moreover, the hierarchy of the displacement modes obtained is not conveniently justified.

In general, the orthogonalization procedures are dependent on specific cross-section mechanics, often justified through a metric definition of the function's space without an intimate connection with a physical meaning. Yet, the definition of such criterion should be uniquely defined and underpinned by a strong physical concept from a structural behaviour perspective. Thus, seeking such an uncoupling through successive generalised eigenvalue problems it is not the conceptual idea to pursuit. The concept should be instead to uncouple the solutions of the governing differential equations rather than the equation itself, which would allow to obtain an hierarchical set of uncoupled solutions, having each one of them a precise physical meaning.

Hence, the ability of the formulation to accurately represent the thin-walled structural behaviour relies not only on the selection of appropriate "enriched" approximation functions, but also, and more importantly, on an uncoupling criterion conveniently justified that allows the definition of uncoupled deformation modes.

A higher order thin-walled beam model that assumes the cross-section in-plane rigid but considers the out-of-plane warping and the "membrane" shear deformation is presented in this paper. An efficient procedure for obtaining warping modes is derived from the linear eigenvalue problem associated with the solution of the model's governing equations, representing an alternative procedure to the one presented by the authors in Vieira [53].

The approximation of the displacement field adopted in the formulation of the higher order model considers an interpolation of the displacement components by a set of linear independent functions defined on the beam cross-section domain. As such, the definition of the displacement field is independent from the cross-section mechanical behaviour and hence it can be successfully applied to any cross-section type, allowing to consider all cross-section membrane deformation components.

The non-linear eigenvalue problem associated with the corresponding beam governing equations is reduced to a generalised eigenvalue problem for the in-plane rigid cross-section. The deformation modes are obtained from the linear eigenvalue problem and the conditions of orthogonality between modes associated with distinct eigenvalues are derived. Deformation modes are therefore uncoupled, allowing to separate the different thin-walled structural behaviours. The procedure allows to obtain not only classic deformation modes (without considering any *ab initio* hypotheses), but also higher order warping modes. The shear deformation of the middle surface stems directly from the model formulation given the procedure adopted for the approximation of the displacement field.

The model is applicable to cross-sections with a generic geometry, e.g. open, closed and with more that two non-aligned walls intersecting a cross-section node, since the displacement components are approximated independently without considering restrictions as a result of deformation hypotheses (see Fig. 1).