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Spectral representation-based neural network assisted stochastic structural mechanics

Dimitris G. Giovanis, Vissarion Papadopoulos *

Institute of Structural Analysis and Antiseismic Research, National Technical University of Athens, Iroon Polytechniou 9, Zografou Campus, Athens 15780, Greece

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ABSTRACT

This paper explores the applicability of artificial neural networks (ANN) for predicting the spread of structural response under the presence of uncertain parameters described as random fields. The use of ANN is carried out in combination with Monte Carlo simulation (MCS) for calculating response statistics in stochastic analysis of structural systems using finite elements. To this extent, the ANNs are trained with a few samples, following a conventional MCS procedure and used henceforth to predict the stochastic response for the rest of samples. The basic idea is to achieve a dimensionality reduction of the input ANN training space by using as input vector the random phase angles of the spectral representation method instead of the random variables describing the uncertain input parameters. A further improvement of the efficiency of the proposed approach is achieved by exploiting the uniform distribution of the random phase angles, in order to span efficiently the training space using a latin hypercube sampling (LHS) technique. The advantage of this approach over conventional computation of stochastic response via a standard stochastic finite element-based MCS is the fast and reliable prediction of the required response sample space which can be accomplished at a fraction of computing time and is independent of the size of the finite element model. Numerical results are presented, demonstrating the efficiency and the applicability of the proposed methodology as well as its distinct advantages over existing ANN-based stochastic finite element methodologies (SFEM).

1. Introduction

The impact of uncertainties in the design process of engineering structures is an important field with growing interest. While most of modern design codes rely on partial safety factors calibrated to target structural reliability, stochastic modeling of the uncertainties in the loading, geometrical, and material properties becomes more and more attractive leading to more rational estimations of structural safety and reliability. Several probabilistic structural analysis methods have been proposed in the past, the simplest being the description of the uncertainties by a set of correlated random variables, where each variable represents a material parameter, load factor, or geometrical property. In several engineering applications however, the description of uncertain parameters using random variables can be insufficient. This is due to the fact that certain physical quantities are often expected to vary randomly in space or time. The probabilistic description of such quantities requires the consideration of random fields. This approach is generally referred as stochastic structural analysis [1].

In principle, brute force MCS is the most suitable and easily implemented method to solve the aforementioned problems. Despite its generality, MCS has been used mostly as a means of verifying the accuracy of approximate and less costly procedures due to its usually high computational cost even if structural analysis is accelerated by advanced solution techniques and/or Neumann series expansion methods [2–4]. To alleviate this drawback advanced variance reduction-based simulation methods have been proposed in the context of reliability analysis, in order to reduce the number of MCS required for an accurate prediction of the probability of failure, such as adaptive sampling, importance sampling, line sampling and subset simulation [5–8]. In addition to the aforementioned methodologies, meta-models, such as artificial neural networks (ANN) have been successfully implemented in the framework of reliability analysis leading to cost-efficient yet acceptable predictions of the probability of failure [9–11]. This is due to the associative memory properties featured by these artificial intelligence devices that allow them to become efficient surrogates to the numerical solver of the mechanical model which is repeatedly invoked in the MCS. Following these early approaches, ANN approximations of the limit-state function were proposed in [12,13] combined either with MC or with first and second order

* Corresponding author. Tel.: +30 2107724158.
E-mail address: vpapado@central.ntua.gr (V. Papadopoulos).

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reliability methods (FORM, SORM) for handling the uncertainties. Similar approaches were presented in [11,14–16] where ANN based response surface methods were implemented in order to estimate the reliability integral over the failure domain. Comparative studies of ANN-based reliability analysis with corresponding polynomial approximations of the response surface as well as FORM and SORM methods are presented in [9,11,17,18]. In those papers, the ability of ANN to predict efficiently and accurately enough the reliability of large and complex structural systems is demonstrated. Furthermore it is shown that the advantage of using the ANN approaches is that they adapt efficiently the input/output (I/O) relations allowing for more accurate mappings of the basic random variables than in the corresponding response surface polynomial approximations of the failure functions, especially in complex failure domains [9].

Despite the fact that ANN have been successfully applied in reliability analysis in an increasing number of publications in which uncertainties are described as random variables, similar applications for the cases where uncertainties are described as random fields are very limited. The main difficulty associated with the use of ANN in the context of stochastic finite element analysis (SFEM) is the large number of random variables induced by stochastic processes describing the uncertain system parameters. This feature of SFEM is making ANN inefficient for treating realistic problems due to the large dimensionality of the input vector. In an effort to alleviate this drawback, ANN were used in [19,20] in the context of Karhunen–Loève (KL) expansion for modeling uncertain material properties of one-dimensional random fields. This way a reduced input vector size for the ANN architecture was achieved by considering the random variables that define each eigenvector of the KL decomposition as the training data set. This ANN-based SFEM methodology was used for computing the response statistics (moments and pdf) of simple statically determinate stochastic beams by providing robust ANN estimates of the structural response in the framework of MCS.

In the present study, an alternative to the aforementioned ANN-based SFEM is proposed which is based on the spectral representation (SR) method [21,22] for the description of uncertain system properties. The basic reason for choosing this alternative approach is that it inherits some distinct advantages of SR method over the KL [1,23], namely its optimal convergence in cases that involve relatively short correlation lengths. An additional advantage of this approach, with respect to the KL-based ANN method is that it avoids the computationally expensive solution of the Fredholm integral equation in cases that involve autocorrelation structures for which analytical solutions of the integral equation are not available. In addition, the proposed methodology provides with a useful extension of ANN-based SFEM methods to multi-dimensional stochastic spaces and is formulated in the framework of the multi-dimensional spectral representation method for the modeling of the involved random quantities. A similar to [19] dimensionality reduction of the stochastic variables is achieved by considering directly the random phase angles of the truncated series expansion as the input vector for the ANN training, instead of the random variables describing the uncertain input parameters. The proposed methodology is applied to Gaussian as well as to non-Gaussian random fields in a straight forward manner. Since ANN require a training phase for tuning their parameters, some conventional Monte Carlo simulations are needed at the initial phase of the procedure. For this purpose, the LHS technique [24] is applied in order to effectively span the random space of the phase angles which, according to the spectral representation, are uniformly distributed in the range $[0, 2\pi]$. Thus, the effectiveness of the proposed ANN methodology is further enhanced by the aforementioned uniformly spanned training space. Numerical results are provided demonstrating the efficiency as well as the applicability of the proposed methodology in a stochastic beam as well as a stochastic plane stress finite element system. In addition, the distinct advantages of the proposed approach over KL-based ANN method are showcased.

The remainder of this paper is organized as follows. In Section 2, an overview of the spectral representation method is made. Section 3 contains a brief description of the basics of artificial neural networks. The proposed method is over-viewed in Section 4. Finally, in Section 5 numerical data is presented demonstrating the computational merits of the proposed methodology in terms of the computational performance.

2. Representation of the random fields

In nature, most of the uncertain quantities appearing in engineering systems are non-Gaussian (e.g. material, geometric properties, seismic loads), the Gaussian assumption is often used due to the lack of relevant experimental data and for simple mathematical convenience. It must be noted that this assumption can be problematic in many cases, for example in the case where the Young's modulus is assumed to be a random variable following a Gaussian distribution. In this case negative values for the Young's modulus may occur which have no physical meaning. From the wide variety of methods developed for the simulation of Gaussian stochastic fields, two are most often used in applications: The spectral representation method [22] and the Karhunen–Loève (KL) expansion as a special case of orthogonal series expansion methods [25]. A comparison between these two methods was made in [21]. The results showed that the KL expansion is particularly suitable for the representation of strongly correlated stochastic fields with smooth autocovariance function where only a few terms, corresponding to the N larger eigenvalues, are required in order to capture most of the random fluctuation of the field. On the other hand the spectral representation method, which was preferred in this work, is mostly suitable for the representation of weakly correlated random fields in which a large number of terms in the series expansion is required in order to capture the random fluctuations. In [23] it was presented that if the stochastic field is homogeneous and the observation interval is infinite the KL expansion reduces to spectral representation. Especially, for the case of a finite long field defined in $[-a, a]$ where a is large, it was shown that the eigenvalue is given by the spectral density function.

2.1. An overview of spectral representation method

Spectral representation method expands the stochastic field as a series of trigonometric functions with random phase angles. The simulation formula for a truncated after N_1 terms, one-dimensional homogeneous random field $\widehat{f}(\mathbf{x}_1)$ reads

$$\widehat{f}(\mathbf{x}_1) = \sqrt{2}\sum_{i=1}^{N_1} A_i \cos(\kappa_{1i}\mathbf{x}_1 + \varphi_i) \tag{1}$$

where $\varphi_i (i = 1, \ldots, N_1)$ are independent random phase angles uniformly distributed in the range $[0, 2\pi]$; the frequencies are set to

$$\kappa_{1i} = i\Delta\kappa_1 = i\frac{\kappa_{1u}}{N_1} \quad \text{for } i = 1, \ldots, N_1 \tag{2}$$

where κ_{1u} is the upper cut-off wave number. The coefficients A_i are defined as follows:

$$A_0 = 0 \quad A_i = \sqrt{2S_{f_0}(\kappa_{1i})\Delta\kappa_1} \quad \text{for } i = 1, \ldots, N_1 \tag{3}$$

where S_{f_0} is the power spectral density function which is a real non-negative function of κ_1. The coefficient A_0 is chosen zero such that the temporal mean value averaged over the whole simulation time