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On a nonlinear single-mass two-frequency pendulum tuned mass damper to reduce horizontal vibration

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ABSTRACT

This paper considers a nonlinear single-mass two-frequency pendulum tuned mass damper (TMD) to reduce horizontal vibration. The proposed TMD contains one mass moving along a bar while the bar can rotate around the fulcrum point attached with the controlled structure. Under a horizontal excitation, the single TMD mass has two motions (swing and translation) at the same time and the proposed TMD has two natural frequencies. In comparison with the optimal linear single mass TMD, because of the inherent nonlinearity of the proposed TMD, it has good performance for large vibration. Moreover, the proposed TMD is also less sensitive to the parameter mistuning. The problem is expressed in the non-dimensional equation form. The approximated vibration amplitudes can be obtained by solving a scalar algebraic equation. The numerical simulation is carried out to verify the approximate analysis.

1. Introduction

A TMD, which consists of a moving mass attached to the main structure through springs and dampers, is a well-known device to suppress vibration. The classical linear single mass TMD is simple but still has some limitations such as the narrow band of suppression frequency and the sensitivity problems due to mistuning. There are a lot of efforts to improve the linear single mass TMD by considering multiple TMD, nonlinear TMD or by adding control. In [1–6] and references herein, one can find several methods to optimize the multiple TMD. The multiple TMD is proved to be more effective and less sensitive to mistuning than the single TMD. Adding control to the TMD leads to the concepts of active TMD, hybrid TMD or semi-active TMD. The readers can find some good overviews of active and semi-active controlled systems in [7,8]. Other studies consider the nonlinear TMD [9–13]. The nonlinearity can be added by the cubic spring or by the impact. Pendulum behavior is another source of nonlinearity. In fact, as the natural frequency of a pendulum depends only on its length, it is easier to tune pendulum TMD frequency in practical applications. The three-dimensional motions of the pendulum TMD has been studied in [14–16]. Some other types of pendulum TMD were presented in [17–19]. The nonlinearity of the pendulum also reveals a new type of TMD called Coriolis TMD or Coriolis vibration absorber [20].

In a structure vibrating in one direction, a linear single mass TMD attached to this structure has only one degree of freedom (DOF) and one natural frequency. Increasing the number of DOF of the TMD can improve its performance. An evident approach is to use the multiple TMD. There are a lot of publications on the multiple TMD. However, because the TMD mass has to be divided into many smaller ones, practical applications of multiple TMD approach still have some limitations. Some other efforts construct multi-DOF single mass TMD [21,22].

In this paper, we use the nonlinearity of the pendulum to increase the number of DOF of the TMD. This paper has two novelties. First, it introduces the two-DOF pendulum TMD and shows the natures of two TMD natural frequencies. Second, the paper presents an approximated solution of the system's vibration by solving a scalar algebraic equation. The approximated solution agrees with the accurate solution while is quite convenient for the TMD optimization. The effective and the insensitivity to mistuning of the proposed TMD, the accuracy of the approximated solution are verified by numerical simulation.

2. Problem statement

The concept of a single-mass two-frequency pendulum TMD is shown in Fig. 1.

When the main mass moves horizontally, the TMD mass has swing motion and translation motion simultaneously. The vibration of the main mass therefore can be transferred more to the

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Table 1
Description of symbols in Fig. 2.

Symbol	Description
k, m	Spring constant and mass of main structure
k_d, m_d	Spring constant and mass of TMD
c_d, c_θ	Damping coefficients in rotation and translation directions
θ, u	Rotation angle and translation displacement
l	Distances between the fulcrum and the damper mass in the static condition
f_0, ω, φ	Amplitude, frequency and phase of excitation

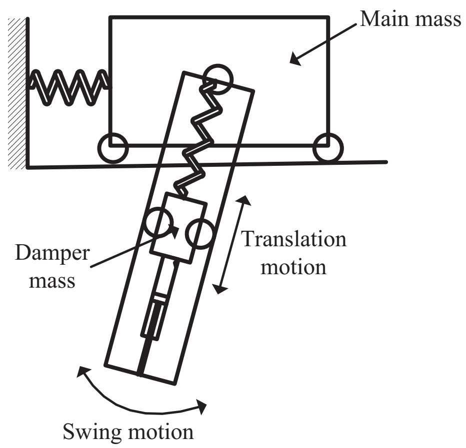

Fig. 1. Concept of single-mass two-frequency pendulum TMD.

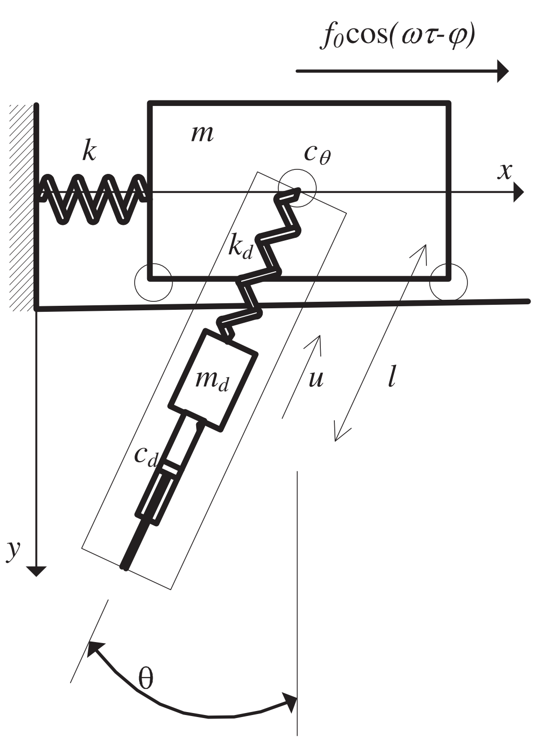

Fig. 2. Symbol used in system modeling.

TMD mass. The swing motion of the TMD mass directly reduces the horizontal vibration of the main mass. The translation motion of the TMD mass acts on the main mass in the indirect way. The translation motion produces the Coriolis force acting on the pendulum while the pendulum swing acts on the main mass. The proposed TMD can have potential applications in vibration control of structures induced by horizontal loads such as seismic, wind or wave loads. Other vibrations can also be controlled by this TMD if there are some mechanisms to convert general vibrations to horizontal vibration.

To write the motion equations, the symbols are shown in Fig. 2 and are summarized in Table 1.

Consider the coordinate system in Fig. 2 and denote the horizontal position of the main mass as x, the position of the damper mass (x_d, y_d) is obtained as

$$x_d = x + (l-u)\sin\theta, \quad y_d = (l-u)\cos\theta \tag{1}$$

The kinetic energy T, the potential energy V, and the energy dissipation function F are

$$T = \frac{m\dot{x}^2}{2} + \frac{m_d(\dot{x}_d^2 + \dot{y}_d^2)}{2}, \quad V = \frac{kx^2}{2} + \frac{k_d u^2}{2} + m_d g(l - u - y_d),$$
$$F = \frac{c_d\dot{u}^2}{2} + \frac{c_\theta\dot{\theta}^2}{2} \tag{2}$$

The system has 3 degrees of freedom: x, θ and u. The Lagrange equations are given by

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial(T-V)}{\partial\dot{x}}\right) - \frac{\partial(T-V)}{\partial x} + \frac{\partial F}{\partial\dot{x}} = f_0\cos(\omega t - \varphi) \\ \frac{d}{dt}\left(\frac{\partial(T-V)}{\partial\dot{\theta}}\right) - \frac{\partial(T-V)}{\partial\theta} + \frac{\partial F}{\partial\dot{\theta}} = 0 \\ \frac{d}{dt}\left(\frac{\partial(T-V)}{\partial\dot{u}}\right) - \frac{\partial(T-V)}{\partial u} + \frac{\partial F}{\partial\dot{u}} = 0 \end{cases} \tag{3}$$

Using (1), (2) in (3) gives

$$(m+m_d)\ddot{x} + kx - m_d(\ddot{u}\sin\theta + 2\dot{u}\dot{\theta}\cos\theta - (l-u)\ddot{\theta}\cos\theta + (l-u)\dot{\theta}^2\sin\theta)$$
$$= f_0\cos(\omega t - \varphi)$$
$$m_d(l-u)(-2\dot{u}\dot{\theta} + (l-u)\ddot{\theta} + \ddot{x}\cos\theta + g\sin\theta) + c_\theta\dot{\theta} = 0$$
$$m_d(\ddot{u} - \ddot{x}\sin\theta + (l-u)\dot{\theta}^2 - g(1-\cos\theta)) + k_d u + c_d\dot{u} = 0 \tag{4}$$

To write the equations in non-dimensional form, some parameters are introduced in Table 2.

The motion Eq. (4) are simplified and rearranged as following non-dimensional form:

$$(1+\mu)\ddot{z}_1 + z_1 - \mu(\ddot{z}_2\sin\theta + 2\dot{z}_2\dot{\theta}\cos\theta - (1-z_2)\ddot{\theta}\cos\theta$$
$$+ (1-z_2)\dot{\theta}^2\sin\theta) = f_n\cos(\beta\tau - \varphi)$$
$$(1-z_2)(-2\dot{z}_2\dot{\theta} + (1-z_2)\ddot{\theta} + \ddot{z}_1\cos\theta + \alpha_1^2\sin\theta) + 2\zeta_1\dot{\theta} = 0$$
$$\ddot{z}_2 - \ddot{z}_1\sin\theta + (1-z_2)\dot{\theta}^2 - \alpha_1^2(1-\cos\theta) + \alpha_2^2 z_2 + 2\zeta_2\dot{z}_2 = 0 \tag{5}$$

in which the dot operator from now denotes the differentiation with respect to normalized time τ. The full nonlinear Eq. (5) are used in the numerical calculations in Section 4. Before moving further, some general characteristics of the proposed TMD can be drawn from (5). To the first order, Eq. (5) can be written as:

Table 2
Symbols used to write the non-dimensional equations.

Symbol	Description
$\omega_s = \sqrt{\frac{k}{m}}$	Natural frequency of the main structure
$\tau = \omega_s t$	Non-dimensional time with time scale $\omega_s{}^{-1}$
$\mu = \frac{m_d}{m}$	Mass ratio of TMD
$\alpha_1 = \frac{\sqrt{g/l}}{\omega_s}, \alpha_2 = \frac{\sqrt{k_d/m_d}}{\omega_s}$	Natural frequency ratios in two directions of TMD
$\zeta_1 = \frac{c_\theta}{2l^2 m_d\omega_s}, \zeta_2 = \frac{c_d}{2m_d\omega_s}$	Damping ratios in two directions of TMD
$z_1 = \frac{x}{l}, z_2 = \frac{u}{l}$	Non-dimensional forms of the displacements
$f_n = \frac{f_0}{kl}, \beta = \frac{\omega}{\omega_s}$	Non-dimensional forms of excitation amplitude and excitation frequency