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Form finding and structural analysis of actively bent timber grid shells

B. D'Amico *, A. Kermani, H. Zhang

Centre for Timber Engineering (CTE), School of Engineering and the Built Environment, Edinburgh Napier University, Edinburgh, UK

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A B S T R A C T

Grid shells are efficient structural systems covering large open spaces with relatively small amount of materials. Also, post forming techniques allow realization of geometrically complex (free-form) shapes by means of standard connection systems. However, due to complexity of the analysis–design process, they are rarely utilized in construction design. In this paper, a 'facilitating' numerical framework is introduced in which, for a given continuous reference shape, a geometrically similar discrete model is found by implementation of a six degree of freedom formulation of the Dynamic Relaxation method, to handle members bending and torsional stiffness. A grid cutting pattern algorithm is introduced, as well as methods to numerically simulate the double-layer construction technique and a novel (single-node) cylindrical joint model. The methods are extensively tested and validated on a range of structures, from 'simple' single-rod cases to more complex, actively bent, grid shell frameworks.

1. Introduction

Free-form grid-shell structures can be formed by connecting short straight beam elements together into nodes thus converting a curved continuous surface in a faceted shell. From a geometrical point of view, the described process deals with complex connection systems: Each element converges to the node of the grid at a different angle, thus non-standard connections (and Computer-Aided Manufacturing process) become inevitable. A 'low-tech' method for building free-form structures using standard (bolted/screwed) connection systems is by bending initially flat elastic rods, such as solid timber planks/laths, to form actual continuous curves. For timber grid shells made of continuous bending members, two sub-categories can be defined [1] differentiating on the geometric parameters assigned to generate a grid on a surface: If screwed laminated timber ribs are arranged following geodesic patterns (shortest curve on a surface for two given points) the planks composing the rib will only be subjected to torsion and bending around the weak axis [2] enhancing the 'allowable' width of the the plank's cross section. This technique was used for the construction of the Hannover Expo pavilion [3]. A different approach was adopted in the design of the Mannheim timber grid shell for the Garden Festival [4]. In this case, it was assumed a constant distance (50 mm) between the consecutive nodes belonging to the same rib, which was built-up with two overlapping laths (double layer technique). Accordingly, the resulting mesh geometry of the grid shell did not follow the geodesic paths (thus, lateral bending occurs as well). However, this second design approach allowed the possibility of assembling the grid shell laid out flat (as a two-way mat of straight continuous rods) and eventually post forming it in a double curved geometry by imposing external displacements under the form of temporary crane-cable systems or adjustable scaffolding [5–7]. With the main grid eventually formed, additional bracing elements can be added to the system enhancing the in-plane shear stiffness of the equivalent shell (Fig. 1). The terms *post formed* [7,8], *actively* or *elastically bent* [9] are usually used to describe such kind of grid shell structures.

2. Simulating the forming process

Since the construction of the Mannheim grid shell, only rarely this (latter) technique has been used. According to Kelly et al: *'The reason for the apparent lack of enthusiasm may stem from the unique challenges associated with the design and formation process'* [10]. Indeed, in order to draw out the post-formed grid shape (and gain information on the internal stress fields) a geometrically non-linear analysis is required to simulate the forming process.

* Corresponding author. Tel.: +44 (0) 131 455 2249.
E-mail addresses: bernardinodamico@gmail.com, b.d'amico@napier.ac.uk (B. D'Amico), a.kermani@napier.ac.uk (A. Kermani), j.zhang@napier.ac.uk (H. Zhang).

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Regardless of the adopted numerical algorithm, the analysis will require the definition of initial parameters to be performed:

- The flat mat geometry (cutting pattern).
- The displacements' vector (boundary conditions).

'To design the Downland grid-shell, physical modelling of the structure was a central element of the design process, which was used to determine the boundary condition for the form-finding model' [5]. The final position of the boundary nodes detected from the 1:30 scale model was used as target to the external imposed displacements and the mat's cutting pattern geometry was (a priori) established to have a rectangular shape. Clearly, any simulation of the forming process will always require the preliminary modelling of scale models.

2.1. A two-step analysis approach

Different authors [7,11–16] addressed the problem of defining purely numerical procedures to speeding up the design process of such structural systems where a built up mat of initially straight elastic rods (usually timber or fiber reinforced polymer) is bent to obtain a form-resistant grid shell structure. Among these, an interesting concept that comes out is that of performing a geometrically non-linear analysis involving the use of a supporting surface on which the mat is 'forced' to bend. In general, the form finding procedure contemplates two consecutive analysis steps where the stress field is generated at the completion of the first step and then is carried forward as *initial condition value* on the second one:

- An initially unstressed (flat) two-way mesh is pulled on the reference surface by means of external axial springs [15] or external applied forces [13]. Alternatively, the mesh is positioned directly on the surface and constrained to slide on it [14].
- With the equilibrium shape found, the mesh geometry exceeding the reference surface is 'deleted' [15] (a cutting pattern is thus found) and translational degree of freedom (DoF) of the boundary nodes are constrained while previous external forces/constraints (shaping the net on the reference surface) are disabled/released, thus the system will assume a new equilibrium geometry, settling down to its final configuration.

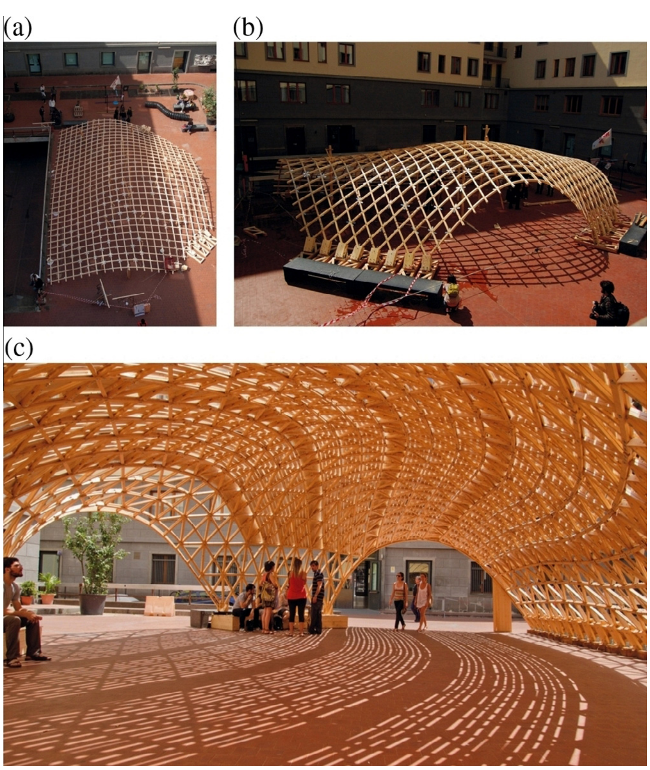

Fig. 1. Toledo timber grid shell in Naples, Italy 2012 [7]: (a) initial flat mat; (b) forming process; (c) complete structure.

Clearly, a two-step analysis scheme allows finding the equilibrium shape that is close to a reference surface which (acting basically as a form-work) can be modelled in accordance to a wide range of design requirements: Harris et al clearly explained how architectural and regulation parameters were driving the shape of the Pods grid shell roof and only in a second design phase *'...a number of trials were made to establish a grid onto the surface'* [17]. Moreover, with such approach there is no need for preliminary scale models since the mesh geometry (cutting pattern of the mat) and boundary conditions are obtained through the first analysis step.

3. Problem statement

Although information on methods involving the use of a reference surface can be found in literature [13–15,18] an effective description for a comprehensive numerical framework, and relative theoretical basis, seems still missing. The aim of the present study is to give a detailed description of the numerical implementation of the introduced methods.

4. Theory

4.1. The Dynamic Relaxation method

The DR is a fictitious time step marching scheme where, the position of the nodes representing the structural system is obtained by iterative numerical integration of the Newton's second law of motion until the entire system settles down in static equilibrium by application of a viscous or kinetic [19] damping term. The method was independently proposed by Day [20] for the analysis of prestressed concrete pressure vessels and Otter et al. [21] although (as noted by Topping and Khan [22]) its concept was already known by Rayleigh. The method has been extensively used for a wide range of structural problems with both geometric and material non-linearities as for instance, the form finding and load analysis of tension structures [23] where it provides more reliable results in terms of solution convergence if compared to the well known iterative matrix schemes with Newton–Raphson method [24]. Moreover, the DR does not require assembling/manipulation of a global stiffness matrix, hence it is relatively easy to implement and is highly suitable for parallel computing [22].

4.1.1. Rotational formulation with DR

The DR method is typically implemented by considering three DoF per node, where each link connecting two nodes can only simulate cable/strut behavior-like. In spite, it has been shown that three DoF schemes are able to simulate bending stiffness [25] if identical second moments of area, around any axis of the elements cross section, are provided. Further, torsional stiffness can be modelled as well by three DoF schemes if (in addition to cross section symmetry requirements) beams with a naturally curved (unstressed) shape are considered [26] and only small deflections are provided to occur. In order to provide a widely applicable method, not restricted to the aforementioned limitations, a more comprehensive Six DoF per node DR scheme (three translational DoF plus three rotational DoF) is here introduced.