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Large torsion analysis of thin-walled open sections beams by the Asymptotic Numerical Method

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ABSTRACT

This paper presents a continuation algorithm based on the Asymptotic Numerical Method (ANM) to study instability phenomena of large torsion of thin-walled open sections beams under various external loadings. The proposed algorithm connects perturbation techniques with a discretization principle and a continuation method without the use of a correction process. In the model, the equilibrium and material constitutive equations are established without any assumption on torsion angle amplitude. In presence of eccentric loads and large torsion context, the right hand side of the equilibrium equations is highly nonlinear and contributes to the tangent stiffness matrix. A 3D beam element having two nodes with seven degrees of freedom is considered in mesh process. Several numerical examples from buckling of thin-walled open sections beams are analyzed to assess the efficiency and the reliability of the method. Comparisons are made with known commercial software. The proposed ANM algorithm is more reliable and less time consuming than other iterative classical methods.

1. Introduction

The structural strength and stability of the majority of laterally unrestrained prismatic or tapered beams are governed by lateral-torsional buckling behavior. In literature, closed-form solutions for the buckling of thin-walled members date back to the end nineteenth century [1–4]. They are detailed in standard books [5–8]. These models developed for small non-uniform torsion has been commonly adopted in most theoretical and finite element works on thin-walled elements with open sections [9–12]. In these studies, the stability analysis of such structures is limited to the determination of buckling loads of perfect structures carried out from solution of the classical eignevalue problem and recourse to design rules is preferred for design necessity. Effects of load height application, Wagner coefficients and boundary conditions on the stability are studied extensively in [13–15]. Application of the boundary element method in behavior and stability of thin-walled structures is investigated in Dourakopoulos [16]. Numerical methods for cross sections characteristics are investigated in [17–20]. Currently, these structures often work in the nonlinear range. Then, a nonlinear analysis is necessary for taking into account large displacements and inherent coupling equations. The nonlinear behavior of thin-walled is more complex than the linear models investigated according to Vlasov's model: (1) the torsion behavior is predominated by the shortening effect proportional to cubic terms of the twist angle, (2) the beam stability is highly dependent on pre-buckling deflection related to the ratio of bending strengths about cross section axes, (3) 3D successive rotations do not commutate and more attention is then demanded and (4) the solution is highly dependent on imperfections and solution control. Kim et al. [21] investigated a finite element formulation with consideration of semi-tangential moments and rotations. Kwak et al. [22] formulat ed a finite element analysis to account for warping effect on the nonlinear behavior of open section beams, by using the Total Lagrangian formulation. Turkalj et al. [23,24] have introduced a correction in the rotation matrix by considering higher order terms for elastic and inelastic behavior. Nonlinear behavior of thin-walled beams with arbitrary cross sections in plasticity and dynamics are studied in [25,26]. Effects of pre-buckling deflection on beam stability are available in [27–32].

Many finite elements models have been proposed for the stability and nonlinear behavior of thin-walled beams with open constant and variable sections in large rotations [33–36].

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The assumption of finite torsion and trigonometric functions are approximated by cubic polynomial functions of the twist angle adopted in these models. These models have been successfully applied for buckling and lateral buckling analysis of thin-walled beams with open sections and improved solutions have been obtained by these models. Mohri et al. [37] investigated a beam finite element model for thin-walled beams with arbitrary cross sections in the context of large torsion without any assumption on the torsion angle amplitude. In the previous numerical models, the nonlinear equilibrium equations are solved by the help of the incremental iterative methods. The equilibrium paths are found by a series of prediction correction procedure according to a known parameter control and a fixed residue. The arc-length methods are more efficient and for this, they are commonly adopted in solution procedure. In the presence of bifurcations and singular points the solution is not unique and depends on the presence of imperfections and their amplitudes. These methods are fastidious and are time consuming in presence of highly coupled equations and systems with large degrees of freedom.

In this paper, the finite element large torsion model for thin-walled beams investigated in Mohri et al. [37] is extended to the case of eccentric loads. Various external loadings are applied on the cross section contour of the beam. Effects of load eccentricities are taken into account in bending and torsion moments without any assumption on the torsion angle amplitude. As a consequence, the left hand side of the equilibrium equations are highly nonlinear. The tangent stiffness matrix depends not only on displacements and initial stresses but also on the applied loads. In solution strategy, we present a new continuation algorithm based on the Asymptotic Numerical Method (ANM) in the solution of the nonlinear system. This algorithm connects perturbation techniques with a discretization principle and a continuation procedure without the use of a correction process [38–41]. This method has been applied successfully in many fields of solid and fluid mechanics [42–49]. Let us remind that in previous works adopting the ANM as solution strategy, the nonlinear terms are due essentially from Green Lagrange tensor. In the present original work, additional nonlinear terms result from the right hand side of the equilibrium, since the vector force is displacement dependant.

The principle of the ANM is simply to expand the unknown of the discretized nonlinear problem in power series with respect to a path parameter "a" allowing transforming the nonlinear problem into a succession of linear ones that admit the same tangent matrix. The interval of validity of the path parameter, $[0, a_{max}]$, is deduced from the computation of the truncated vector series. So, the step lengths are computed a posteriori by an estimation of a_{max}. By using the evaluation of the series at $a = a_{max}$, we obtain a new starting point and define, in this way, the ANM continuation procedure. In this algorithm, only one tangent matrix triangulation, at the current point, is needed to compute the terms of the series. This algorithm is robust and often leads to a fast convergence by increasing the truncation order.

In this paper we propose some ideas that enable us to adapt the series method to strongly non-linear problems. In this case, to be able to compute a large number of terms of the perturbation series, we suppose that the trigonometric functions $(c = \cos(\theta_x) - 1)$ and $(s = \sin(\theta_x))$ are then included as additional variables in the model [40]. The treatment of this high nonlinearity will be facilitated in the ANM algorithm by introducing some additional quadratic relations in the form $dc = -s d\theta_x$ and $ds = (c + 1) d\theta_x$, obtained by a simple differentiation of the functions c and s.

The paper is organized as follows: in Section 2, we present the nonlinear model of thin-walled beams with open sections and the derivation of the equilibrium equations of these structures. The Section 3 is devoted to the proposed ANM continuation algorithm. Several numerical examples on nonlinear behavior and stability of thin-walled open sections beams are analyzed to assess the efficiency and the reliability of the method are illustrated in Section 4. It is proven that the proposed ANM algorithm is more reliable and less time consuming than the classical iterative methods. Conclusions close the paper at Section 5.

2. Nonlinear structural model

2.1. Nonlinear kinematics of the thin walled beam element with open section

Consider a 3D open section straight thin-walled beam element of beam length L and cross section $A(x)$ as illustrated in Fig. 1a. The adopted reference system is $(Gxyz)$ of center G and of rectangular axes Gx, Gy and Gz such that Gx is the initial longitudinal axis, Gy and Gz are the first and second principal bending axes respectively. The co-ordinates of shear center C located in Gyz plane are (y_c, z_c), those of a point M on the section $A(x)$ are (y, z, ω), where ω is the sectorial co-ordinate which characterizes the warping of the section at point M for non uniform torsion (see Fig. 1b) [5].

We assume in this model that the beam has an elastic behavior and the local and distortional deformations are neglected. In the framework of large displacements, large twist angles and small deformations, the displacements u_M, v_M, w_M of a point M are expressed by the following nonlinear relations [36,37,29]:

$$\begin{aligned} u_M &= u - y(v' + cv' + sw') - z(w' + cw' - sv') - \omega\theta'_x \\ v_M &= v - (z - z_c)s + (y - y_c)c \\ w_M &= w + (y - y_c)s + (z - z_c)c \end{aligned} \tag{1}$$

where u is the axial displacement of G, v and w are the displacements of shear point C in y and z directions, θ_x is the torsion angle and the two variables c and s which are defined by the following trigonometric functions: $c = \cos(\theta_x) - 1$ and $s = \sin(\theta_x)$. The symbol $(.')$ in Eq. (1) denotes the derivation with respect to the co-ordinate x. The Vlasov's linear model [5] can be recovered from Eq. (1) by approximating the trigonometric functions c and s by 0 and θ_x respectively and using linear assumptions. Since the model is concerned with large torsion, the functions c and s are conserved without any approximation in both theoretical and numerical analyses.

The considered nonlinear Green strain tensors, taking into account the large displacements, membrane effect, bending, nonlinear warping effect, has the following components:

$$\begin{aligned} \epsilon_{xx} &= \epsilon - yk_z - zk_y - \omega\theta''_x + \frac{1}{2}R^2\theta'^2_x \\ \epsilon_{xy} &= -\frac{1}{2}(z - z_c + \frac{\partial\omega}{\partial y})\theta'_x \\ \epsilon_{xz} &= \frac{1}{2}(y - y_c - \frac{\partial\omega}{\partial z})\theta'_x \end{aligned} \tag{2}$$

where ϵ is the membrane component, k_y and k_z are beam curvatures about y and z axes and R is the distance between the point M and the shear center C, expressed by:

$$\begin{aligned} \epsilon &= u' + \frac{1}{2}(v'^2 + w'^2) - \psi\theta'_x \\ k_y &= w'' + cw'' - sv'' \\ k_z &= v'' + cv'' + sw'' \\ R &= \sqrt{(y - y_c)^2 + (z - z_c)^2} \end{aligned} \tag{3}$$

where $\psi = y_c(w' + cw' - sv') - z_c(v' + cv' + sw')$ is the variable associated with membrane component. In Eq. (2) the quadratic contribution of the axial displacement u_M in $\epsilon_{xx}, \epsilon_{xy}$ and ϵ_{xz} has been neglected.